Effective Fertility: A New Measure of Surviving Children to Analyze Fertility Decline

Anup Malani, Ari Jacob*

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Abstract

The consequences of fertility decline flow primarily from reduction in the number of surviving children, i.e., both births and survival, not simply from reductions in births. Therefore, neoclassical economic theories that attempt to explain fertility decline (e.g., quantity-quality models) hinge on preferences for surviving children, not births per se. This paper provides a new measure of surviving children. Effective fertility rate (EFR) is the product of the current fertility rate and the projected probability that a child survives until some age A. This age A could be between 15-49 for a measure (EFR_R) that tracks reproductive potential, or 15-65 for one (EFR_L) that tracks income and taxes paid. While EFR_R can be approximated by existing demographic concepts such as the net reproductive rate or the ratio of total fertility rate (TFR) to the replacement fertility level, EFR_L is poorly approximated by existing concepts.

We use three data sets to shed light on EFR across time and geography. First, we use data from 165 countries between 1950-2019 to show that one-third of the global decline in TFR during this period did not change EFR_L, suggesting that a substantial portion of fertility decline merely compensated for higher survival rates. Focusing on the change in EFR_L, at least 40% of variation cannot be explained by economic factors such as income, prices, education levels, structural transformation, an urbanization, leaving room for explanations like cultural change. Second, using historical demographic data on European countries since 1750, we find that there was dramatic fluctuation in EFR_L in Europe around each of the World Wars, a phenomenon that is distinct from the demographic transition. However, prior to that fluctuation, EFRs were remarkably constant, even as European countries were undergoing demographic transitions. Indeed, even when EFRs fell below 2 after 1975, we find that EFRs remained stable rather than continuing to decline. Third, data from the US since 1800 reveal that, despite great differences in mortality rates, Black and White populations have remarkably similar numbers of surviving children over time.

*Malani: University of Chicago and NBER; Jacob: University of Chicago. We thank Ryden Iwamoto and Dorian Abbot for helpful discussions.

Introduction

Changes in fertility can be grouped into two buckets: changes that hold the number of surviving children produced by households constant and changes that alter number of surviving children per household. The first bucket merely compensates for changes in child survival rates. It has few first-order effects, positive or negative, on human welfare. While classical economic theory, such as the Malthusian model, can explain first-bucket changes, neo-classical theories focuses on changes in the number of surviving children.

In contrast, the second bucket of changes in the number of surviving children is hypothesized to have serious consequences. One view is that additional surviving humans use up more resources, holding resources constant. A different view is that more surviving humans may increase resources by generating innovations. Moreover, more adults means funding for a pay-as-you-go safety net (formal, public insurance) or old-age care by (grown) children (informal private insurance). This second bucket is also the primary subject of neo-classical models of family size. The path-breaking Becker-Lewis-Barro class of models (Becker, 1960; Becker and Lewis, 1973; Barro and Becker, 1989a) put the number of surviving children into the utility function. Similarly, the children-as-capital models presume children survive, else they are a poor store of capital.

In this paper, we present a novel measure of surviving children. We define a concept we call "effective fertility rate at time t for age A" (EFR(A, t)) to be: the number of children born to each female at time t × the probability that they survive until age A. This probability of survival is based on future mortality rates, which may require the use of mortality projections. For example, EFR$(65, t)$ is the fraction of children born today (t) who will live until they are 65.

We highlight two variants of EFR that are uniquely relevant to welfare and policy. First, we define EFR_R as the number of daughters that live long enough to reproduce, between ages 15 and 49. This focuses on daughters, not all children, because only females reproduce. Because a child need not live until age 49 to reproduce, we approximate EFR_R by taking the average of EFR over all reproductive ages (15-49). Second, we define EFR_L as the number of children born today who will live long enough to earn labor income. We approximate this by taking the average of EFR over all working ages (15-65) To calculate these EFRs, we use current country-level TFR and past and projected age and country specific survival rates, typically obtained from UN data.

Conveniently, the first of these variants, EFR_R, can be approximated with existing concepts in demography. One is net reproductive rate (NRR), which is defined as the number of children a female at time t will have throughout her reproductive period, accounting for mortality rates, and is regularly estimated by the UN. While NRR is focused on female survival only until age 45 (rather than 49) and uses current rather than future mortality rates (unlike EFR), multiplying NRR by two (to account for sons) is a reasonable approximation for EFR_R where mortality rates are already low or stable.

The other concept is the ratio of TFR to replacement level fertility (RLF). While TFR (defined as the sum of age-specific fertility rates of females from ages 15-49) does not account for mortality, RLF (defined as the number of children each female must have to ensure a population yields the same number of reproductive-age females it currently has) does account for maternal mortality. Although demographers talk about TFR and RLF separately, we will show that the ratio of TFR to RLF, times two[1], is a reasonable approximation for EFR_R. This is true even though RLF—like NRR–uses current mortality rather than projected mortality.[2]

These approximations are less adequate for EFR_L because they do not account for the (higher) mortality of males and because they do not account for the (higher) mortality between ages 50 and 65.

One advantage that EFR has over TFR is that it easier to determine replacement level effective fertility than replacement level (total) fertility rates.[3] The replacement level for effective fertility is 2 across all countries. This contrasts with replacement level for total fertility, i.e., the level of TFR required to maintain a stable population over the long run. Replacement level for total fertility varies across countries depending on country mortality rates because TFR does not account for mortality. In contrast, EFR directly account for mortality rates, so replacement level is the same across countries.[4] As such, EFR has more content as a summary statistic than does TFR.

To demonstrate the value of EFR calculations, we conduct two exercises. One exercise decomposes changes in TFR—which is the focus of most news reports about fertility decline—into changes that hold EFR constant and changes that lower EFR. In doing so, we are able to assign TFR changes to the first or second bucket of fertility changes. Using data from 165 countries from 1950-2019, we find, first, that on average 36% percent of the global fertility decline during this period merely compensated for increases in survival rates, i.e., held EFR constant. This tends to be larger in regions like Africa, which have experienced more substantial declines in mortality. Over time, we find that fertility declines until 1990 typically maintained EFR, while declines since then have lowered EFR.

We also perform the decomposition using data on Western European countries since 1750 and on Black and White populations in the United States since 1800. In Western Europe, we observe remarkable cycling of EFR over time, and especially around the two World Wars. However, prior to the World Wars, EFR ranges from 2-3 and was remarkably consistent, even as countries were undergoing the demographic transition. Moreover, even as EFRs fell below 2 after 1975, those EFRs remained remarkably stable across most sample countries rather than continuing to decline. In the United States data, we find that, despite dramatically higher rates of mortality among Black populations, the number of surviving children are remarkably similar across the two racial groups.

Our second exercise estimates how much changes in EFR can be explained by economic forces, versus non-economic factors, such as culture. Specifically, we correlate changes in

[1] Two is a crude approximation, both for the adjustment to NRR and to TFR/RLF. A more accurate approximation would use 1+SRB, where SRB is sex ratio at birth.

[2] Where projections are unavailable or unreliable, the use of current mortality rates is an advantage of NRR or TFR/RLF approximations. However, as we will show, one can also calculate EFR_R or EFR_L with current mortality rates.

[3] We put total in parentheses because people do not usually quality "replacement level fertility" by noting it identified the TFR that achieves replacement.

[4] To be fair, this is only true if countries have equivalent sex ratios at birth and age-specific mortality rates by sex. If a country has, e.g., a high sex ratio (defined as males over females), then replacement effective fertility rate is above 2 because one needs more children to have exactly 1 daughter. We do not highlight this as a weakness of the replacement effective fertility rate because this is also a weakness of replacement level (total) fertility rate.

EFR with purchasing power-adjusted per capital GDP (reflecting income and price changes), share of population employed in agriculture (reflecting demand for children as labor), share of population in cities (reflecting the cost of housing), and educational attainment (reflecting returns to education). We find that factors other these economic forces can explain at least 40% of changes in EFR. (By contrast, they only explain 25% of changes in TFR, suggesting that economic forces explain the number of births better than the number of surviving children.) To the extent that the residual variation reflects factors such as culture, we infer that there is a substantial influence social cues play in driving EFR.

Section 1 reviews literature related to EFR. It surveys literature on demographic concepts that measure both fertility and mortality, and reviews empirical tests of neoclassical economic theories of family size. Section 2 defines EFR(A, t), explains our method for calculating it, shows how well NRR and TFR/RLF approximates EFR_R and poorly they approximate EFR_L, verifies EFR(A, t) as a projection of future age-specific populations, and compares EFR to measures of fertility preferences. Section 3.1 decomposes changes in TFR into EFR-preserving and EFR-reducing components. Section 4 correlates EFR with economic and non-economic forces. Section 5 concludes with a discussion of limitation of EFR and directions for future research.

1 Literature review

In this section, we discuss the relevant demographic and economics literature decline. In the demographic literature, we first discuss several demographic concepts that can be used to approximate EFR_R (but not EFR_L). Because the demographic transition is the biggest driver of population growth and stabilization, i.e., the rise and fall in the number of surviving children per female, we will discuss the relevant components of that literature. In the economics literature, we highlight that economic theories of fertility actually focus on surviving children. We then review empirical tests of those theories and the relationship between mortality and fertility.

1.1 Relevant demographic literature

Net reproductive rate. As we shall show, net reproductive rate (NRR) is the demographic concept most closely connected to what we call reproductive effective fertility rate (EFR_R). (NRR is a poor approximation to labor EFR in lower income countries.) NRR measures the number of daughters a mother is likely to have accounting for the probability that the *mother* survives through her reproductive years. By contrast, EFR_R counts the number of daughter born today and accounts for the probability that the *daughter* survives through her reproductive years.) An NRR of 1 corresponds to replacement level fertility (Preston et al., 2000).[5]

[5] Although NRR is regularly reported by the UN (United Nations, Department of Economic and Social Affairs, Population Division, 2024), it is not as widely used a measure of fertility as total fertility rate (TFR). A fundamental difference between NRR and TFR, however, is that the former accounts for mortality rates of the mother while the latter accounts for no mortality rates. The gross reproductive rate (GRR), which measures the number of daughters that a mother is likely to have through her reproductive years under

The paper closest in spirit—from an empirical perspective—to the present paper is Shen et al. (2023), which shows most changes in NRR have been driven by changes mortality rates. This work is similar to our decomposition of changes in TFR into two buckets: one that compensates for mortality declines but holds number of surviving children constant and one that changes the number of surviving children. There are two ways that we extend Shen et al. (2023), other than the trivial difference that we focus on TFR (and total children) rather than NRR (and only daughters). First, we also estimate the fraction of variation in surviving children that is attributable to economic factors (income, demand for children, cost of children). Second, we extend the sort of analysis in that paper to a larger number of countries or time periods. For example, we include more pre-1950 data on European nations and subnational data on Black and White populations in the US.

Replacement level fertility. Replacement level fertility (RLF) is defined as the number of children a female must have over her reproductive life span in order to exactly replace herself and her partner in the population, accounting for child mortality until children are themselves of reproductive age. Unlike NRR, but like EFR_R, RLF focuses on the mortality of offspring. (Like NRR, RLF can be a poor approximation of EFR_L, which accounts for future labor produced by offspring.) Just as this paper examines how EFR varies across countries, several papers examine how RLF also varies across countries (e.g., Espenshade et al., 2003; Gietel-Basten and Scherbov, 2019).

If TFR is greater than RLF, then a population will grow in the long run, and vice versa. The main substantive difference between RLF and EFR_R is that RLF only provides information on whether a population will grow or shrink. By contrast, EFR_R indicates how much a population will grow or shrink. As we shall show, it is possible to examine the ratio of TFR to RLF to approximate EFR_R and show how much a population will grow or shrink. However, only Espenshade et al. (2003) explicitly calculate this ratio, though only for the period 1995-2000. Importantly, this paper is the first to connect this ratio to the the number of surviving children each female has.

Family size and ideal/desired fertility. There is a literature on family size that generally addresses two questions orthogonal to this paper, but includes one paper that is closely related to the present one. Family size, defined as the number of children in a family. It could be interpreted as capturing the number of children surviving until reproductive age, though that is not the goal of measuring family size not always how family size is interpreted. The first, main question in this literature concern the impact of family size on outcomes, such as education. The second question concerns fertility preferences, i.e., the ideal number of children a female would like to have. This inquiry does not concern child survival. Indeed, there is a very large literature on ideal fertility and it's causes (e.g., Pritchett, 1994; Freedman, 1997; Bongaarts, 2003; Trinitapoli and Yeatman, 2018).

The one paper that discusses family size in a spirit connected to our discussion of surviving children is Lam and Marteleto (2008). That defines family size as a product of fertility rates and child mortality rates, using a formula similar to Eq. (3). Moreover, that paper examines how family size changed through the demographic transition. In doing so, it touches on some

the assumption that the mother survives through them, is a more direct analogue to TFR, differing only in units. GRR measures daughters per reproductive age female, while TFR measures number of children per reproductive age female.

of the ideas that we do when decomposing fertility changes into those that compensate for changes in mortality (but hold number of surviving children constant) and those that change the number of surviving children. In other words, Lam and Marteleto (2008) uses a version of Eq. (21) that has percent change in EFR on the left-hand side to measure the impact of the demographic transition on family size.

Demographic transition. There is a vast literature on the demographic transition, which describes the process by which countries go from high mortality and fertility rates (stage 1), to low mortality and high fertility rates (stage 2), to low mortality and falling fertility rates (stage 3), to low mortality and fertility rates (stage 4). These stages correspond to population growth in stage 2, slowing growth in 3, and population stabilization in 4. To the extent that mortality are slow to change, they also correspond to stable EFR in stage 1, rising EFR in 2, falling EFR in 3, and stable EFR in 4.[6]

The literature on the demographic transition emphasizes the link between the transition and population levels or growth (e.g., Bongaarts, 2009). However, aside from Lam and Marteleto (2008) (discussed above) we could not find many references to the relationship with surviving children. An important difference between the relationship to total population and the relationship to EFR is timing of mortality rates. Population effects depend on fertility and, largely, current death rates at older ages. By contrast, EFR effects depend on birth rates and future death rates at younger ages (Lam and Marteleto, 2008).

The demographic transition spurred a large literature on what caused the demographic transition (e.g., Galor, 2012a). An important section of this literature looked at proximate rather than root economic or cultural causes. By proximate causes, we mean whether declines in mortality caused changed in fertility or whether fertility declines coincidentally lagged mortality declines (e.g., Ben-Porath, 1976; Palloni and Rafalimanana, 1999; Narayan, 2006; Reher and Sanz-Gimeno, 2007; Angeles, 2010; Poppel et al., 2012; Reher and Sandström, 2015; Reher et al., 2017; Shapiro and Tenikue, 2017; Marco-Gracia, 2021; Bhattacharya et al., 2023).[7] The weight of the evidence seems to favor the former view.

There is an important distinction between the goals of these just-referenced papers and the present one. Those papers are trying to determine whether mortality decline caused fertility decline. In the main, the present paper is mainly an accounting exercise that tracks how the number of surviving children (per female) changed over time and the extent to which the changes were driven—in a measurement sense—by the fertility component or the mortality component of the way surviving children are calculated.[8] Our goal is not to

[6]This demographic transition is sometimes called the "first" demographic transition in a smaller literature that discusses the "second" demographic transition, which refers to the period when birth rates fall below death rates, a precursor to population decline. (Sometimes the second demographic transition is simply called stage 5 of the first demographic transition.) This second transition corresponds to the number of surviving children or EFR falling below 2, another linkage that we could not find in the literature.

[7]There is also a literature on whether declines in fertility increases child survival by, e.g., reducing sibling competition for resources (e.g., Lawson et al., 2012).

[8]It is worth highlighting Angeles (2010), which separately regresses TFR and net reproductive rate (NRR) on mortality rates. Importantly, NRR captures mortality and—we shall show—approximates surviving children, on mortality rates. The paper concludes lower mortality significantly lowered TFR, but not NRR, i.e., mortality affected fertility but not a proxy for reproductive EFR. This is compatible with a view that households have preferences over surviving children, not fertility per se; so that mortality may affect fertility to achieve a desired number of surviving children, but mortality does not directly affect the desired level of

determine a causal flow from mortality or surviving children to fertility, but to see how much a decline in fertility in effect compensated for mortality versus reduced EFRs.

1.2 Relevant economics literature

Canonical economic theories that try to explain household fertility choices model parents as caring not just about births, but about the number of surviving children (e.g., Becker, 1960; Becker and Lewis, 1973; Willis, 1973; Easterlin, 1975; Becker and Tomes, 1976; Barro and Becker, 1989b; Becker et al., 1990; Galor and Weil, 1996, 2000; Boldrin and Jones, 2002; Doepke, 2004; Greenwood et al., 2005). This is true even for models that nominally posit that utility is a function of the number of children and other consumption, because the justification they provide for why kids enter parents' utility is that children are a durable good or provide future labor or income.

Because economic models tend to focus on the fall in fertility over the last 2 centuries, during which the Industrial Revolution took place and incomes rise, economic models tend to fall into two groups.[9] One group focuses on the relationship between income and fertility. These models tend to focus on the time cost of raising children and the substitution effect of raising hourly wage. The other group tends to focus on the skill-biased technological change and how that caused parents to shift budgets away from quantity towards quality of children. Jones et al. (2010) has a terrific review of both groups of models and critical assumptions.

There is empirical work associated with these two groups of models that focuses on fertility, even though the models themselves are often justified by claims about the value of surviving children. The first group includes papers that examine the relationship between income and fertility (rather than surviving children) (e.g., Becker, 1960; Jones et al., 2010). And the latter group has papers that examine the interplay of income, fertility, and education of children (e.g., Rosenzweig and Wolpin, 1980; Hanushek, 1992; Schultz, 1997; Angrist et al., 2010; Black et al., 2005; Becker and Tomes, 1976).[10] Two exceptions are Doepke (2005) and Murphy et al. (2008). Although those examine surviving children, that is not their focus and they only examine survival until age 5 or 10.

This paper is more closely related to economic models and empirical analysis of the demographic transition. Delventhal et al. (2021) documents the timing of the transition across countries using the timing of trend breaks in mortality and then fertility. Importantly, Galor (2012a) provides a simple model of the transition that, condition on the desired number of surviving children, parents alter fertility to exactly offset change in mortality.[11] In addition, there is a sizable literature examining the effect of changes in mortality on fertility (e.g., Schultz, 1993; Kalemli-Ozcan, 2002; Lorentzen et al., 2008; Angeles, 2010; Canning et al., 2013). These papers provide evidence on the (negative) sign of the relationship or the specific slope of the relationship. To our knowledge, none examines the resulting number of surviving children and how that has evolved over time.

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surviving children (Galor, 2012b).

[9] Easterlin (1975) is a notable exception.

[10] See also Doepke et al. (2023) for a review of the empirical literature.

[11] See also Cigno (1998) and Atella and Rosati (2000).

2 Definition and comparison to existing concepts

In this section, we do five things. First, we define EFR and show how we calculate it. Second, we motivate EFR from a notion of the value of human lives. Third, we validate EFR by successfully using it to predict age-specific populations. Fourth, we use formulas to compare EFR to related concepts in demography, namely NRR, RFR, and desired fertility. Fifth, we compare EFR to fertility preferences.

2.1 Definition of EFR

Our objective is a measure of the number of children born each year *who will survive*. This number will depend on the age to which we say children must survive, on the time of our calculation because survival rates vary by time, and the units in which we want to report the number. Accordingly, we define an effective fertility rate at time t for age A ($\text{EFR}(A,t)$) as the expected number of children born at t who will survive until age A, per reproductive age female, all times 35. If $B(t)$ is the total number of births at time t,[12] $N(t)$ is the population at time t, $N_{fR}(t)$ is the number of females of reproductive age at time t, $m(a,s)$ is the probability of death at age a in year s, and $S(A,t)$ is the probability at time t of surviving until at least age A, then

$$\text{EFR}(A,t) = 35 \cdot \frac{B(t)}{N_{fR}(t)} \prod_{a=1}^{A} [1 - m(a, t+a)] = 35 \cdot \frac{B(t)}{N_{fR}(t)} S(A,t) \tag{1}$$

The reason we multiply the fraction by 35 is that we want to make EFR comparable to TFR, which is frequently used to measure fertility, in part because of the intuitive appeal of a saying that a coupled female must have approximately 2 children to keep population stable. Although TFR measures the number of children a female at t will have over her reproductive life,[13] it is measured using cross-sectional data at time t, i.e., it sums birth rates for women age 15, for women age 16, to women age 49, all at time t. This is approximately 35 times the ratio of total births to the number of reproductive age females:

$$\text{TFR}(t) = \sum_{a=15}^{49} \frac{B(a,t)}{N_f(a,t)} = \sum_{a=15}^{49} b(a,t) \approx 35 \cdot \frac{\sum_{a=15}^{49} b(a,t)}{\sum_{a=15}^{49} N_f(a,t)} = 35 \cdot \frac{B(t)}{N_{fR}(t)} \tag{2}$$

As a result, we can can not only compare EFR to TFR, we can approximate EFR with TFR and survival rates[14]:

$$\text{EFR}(A,t) \approx \text{TFR}(t) S(A,t) \tag{3}$$

[12] The total births in time t is equal to the crude birth rate $\text{CBR}(t)$ at time t (defined as the number of live births divided by population) times the total population $N(t)$ at time t.

[13] TFR is defined as the average number of children a woman would have over her childbearing years based on current birth rates. It is calculated by summing age-specific fertility rates (ASFR) for ages 15-49. If ASFR are calculated in 5-year age bins, then the sum of AFSRs are multiplied by 5. Note that the TFR calculation is not the same is total number of children born to females of reproductive age at time t because TFR takes a simple sum of ASFRs rather than a weighted sum that accounts for the number of women in each age bin.

[14] We consider the empirical differences in EFR calculations when using the formula in Eq. (1) versus Eq. (3) in Appendix C.

The EFR formula and approximation above will typically be applied in a given location, and vary across locations. Likewise, survival rates vary across females and males and sex ratios at birth differ across locations. For simplicity we do not account for sex differences in mortality rates and sex ratios at birth, but it is easy to modify the formula to account for them.

The survival rate that is most relevant to EFR calculations depends on our goals. If we are measuring reproduction, then the relevant survival rate is that of female children. If we are measuring labor force and both sexes work equally, then the relevant survival is the adult survival rates.

A related question is what age A should we use to measure survival rates for EFR. For example, if the goal is to determine the number of working children a person will have, one might be tempted to use $A = 65$ if the units are years, because a person typically works until age 65. But a person can be valuable for work even if they do not live to 65. A solution is to take simple average of EFR across a relevant age range, which we will indicate using the angle brackets ($\langle \cdot \rangle_{[\underline{a},\bar{a}]}$ for range $[\underline{a},\bar{a}]$).

$$\langle \text{EFR}(A,t) \rangle_{15,65} = \sum_{A=15}^{65} \frac{\text{EFR}(A,t)}{51} \approx \text{TFR}(t) \frac{\sum_{A=15}^{65} S(A,t)}{51} \tag{4}$$

To distinguish $\text{EFR}(A,t)$ and $\langle \text{EFR}(A,t) \rangle$, we drop reference to a specific age A when we refer to EFR.

Building on this insight, we define two more specialized, versions of EFR that would be relevant for specific applications. If one is concerned with future reproductive potential, EFR should focus on the number of daughters born and their survival:

$$\text{EFR}_R(t) = \langle EFR_f(A,t) \rangle_{15,49} = \sum_{a=15}^{49} \frac{\text{EFR}_f(A,t)}{35} \approx \frac{\text{TFR}(t)}{1 + SRB(t)} \frac{\sum_{a=15}^{49} S_f(A,t)}{35} \tag{5}$$

where R subscript indicates reproduction, subscript f indicates female, and S_f is the survival probability for females. Note that the units of EFR_R are daughters per female. If one is interested in the size of the future, working population, then

$$\text{EFR}_L(t) = \langle EFR(A,t) \rangle_{15,65} = \sum_{a=15}^{65} \frac{\text{EFR}(A,t)}{51} = \text{TFR}(t) \sum_{a=15}^{65} \frac{S(A,t)}{51} \tag{6}$$

is the appropriate EFR. Here, L subscript designates labor. The units for this calculation is the number of children of both sexes.

One might be concerned that EFR_R and EFR_L measure the number of daughters or the number of children, respectively, but not all daughters have equal number of children when they later reproduce and not all children may work as adults. One could account for this by multiplying EFR_R by the probability of reproduction or EFR_L by the probability of working. Because this adjustment is mainly important when these probabilities are changing over time, and the derivation of this adjustment entails demographic concerns distinct from those required to estimate survival, we leave these adjustments for future research.

Because our measures of EFR require a reasonable amount of data and taking averages, we examine simple approximations for these measures that are less data and calculation intensive in the appendix. Our measure of EFR requires both projections of future survival and requires taking averages of those survivals across ages. One could use approximations that relax each of those requirements. For example, one could use current survival rates across age rather than future survival rates across time. Or calculate survival probability until a mid-range $\tilde{A} \in [\underline{a}, \bar{a}]$—either the average age in a range or mid-point of that range—rather than averages of the survival probability across that range. Or both. In the appendix, we show that single approximations are reasonable. Moreover, approximating future survival with current survival lead to EFR approximation errors of 1/3 of a child or less, with the errors unsurprisingly concentrated in countries with rapidly falling mortality rates.

Finally, we note an advantage of EFR over TFR as a measure of fertility: it is easier to determine whether an EFR is above replacement level. Because TFR does not capture mortality rates, and mortality rates affect the probability a daughter survives until she is herself old enough to reproduce, one cannot readily infer from TFR whether a country has high enough fertility to maintain a stable population. Instead, one must compare TFR to so-called the replacement fertility level, which captures the fertility required to replace an existing population given prevailing mortality rates. Moreover, RLF varies across countries. While it is commonly said that a RLF of 2.1 constitutes replacement, actual RLF is often substantially higher historically and in lower income countries with higher mortality rates (Espenshade et al., 2003). By contrast, the replacement level EFR_L is always 2. Replacement EFR_R is also very nearly 2. The number of daughters required to replace the population is equal to one. If SRB is the sex ratio (i.e., ratio of boys to girls) at birth, then the number of daughters per every 2 children born is 1/(1+SRB)[15] (This is equal to 1/2 only when SBR = 1.) So, the EFR_R required for replacement is actually 1/(1+SRB) $\approx$ 2. However, this source of variability in replacement EFR_R is not an advantage for TFR: replacement fertility level must also make an adjustment for sex ratios if it aims to measure the number of daughters a reproductive age female has.

2.2 Data and methods

2.2.1 Data

To calculate EFR, and for our subsequent analysis, we create a data set using 5 categories of data: fertility rates, births by sex, population by age group and sex, life tables by sex, fertility preferences, and economic indicators. Data sources for each of these categories are given in Table 1 for three different applications: country × year data for 165 countries from 1950-2019, select country × year data for select European countries from 1750-1950, and Black and White populations × year for the Unites States from 1800-present. Note that the European data does not include data on all countries starting in 1750, and the US data does not include data on Black populations until 1900. The GDP per capita expressed in constant 2011 International Dollars using 2011 purchasing power parity (PPP) (Bolt and van Zanden 2023).

[15] If SRB $= M/F$, where M is number of males and F is number of females at birth, then $F/(F + M) = 1/((F + M)/F) = 1/(1 + SRB)$.

Table 1: Sources of data

Topic	Country x year, 1950-2019	Select European country x year, 1750-1950	US Black and White populations 1800-present
Births, population by age	United Nations (UN) World Population Prospects	Human Mortality Database	
Fertility rate	United Nations (UN) World Population Prospects	[calculated]	CDC Vital Statistics 2016, 2021 (post 1980), Haines 2008 (pre 1980)
Life tables	United Nations (UN) World Population Prospects (including projections)	Human Mortality Database, United Nations (UN) World Population Prospects (only projections)	Haines 1994, Hacker 2010
Fertility preferences	Demographic Health Surveys	n.a.	n.a.
GDP	Bolt and van Zanden 2023	n.a.	n.a.
Education	Barro and Lee 2013	n.a.	n.a.
Agriculture share of employment	Wingender 2014	n.a.	n.a.
Urbanization rate	World Bank 2024	n.a.	n.a.

2.2.2 Calculation of EFR

We shall explain our calculations focusing on the application to all countries the UN reports from 1950-2019. Calculations for the other two applications—select European countries before 1950 and Black and White populations in the US from 1900—will largely follow the same methods. When they differ, we will highlight the differences in footnotes.

Calculating EFR requires three categories of demographic information: total fertility rates (TFR) and survival probability by age. We pull TFR directly from UN data.[16] We calculate survival probabilities for each age from life tables. For years prior to 2024, survival probability is calculated as one minus the number of deaths d_x between age x and $x+1$ divided by the number of persons surviving l_x to age x: $1-(d_x/l_x)$. For years after 2024, we UN projections based on the method described in Li et al. 2013 and Lee and Carter 1992.[17]

We calculate EFR in two steps. First, for each birth year t and age A combination in country j, we calculate $S_j(A,t)$ by taking the product for ages $x = 1, ..., A$ of $1 - q_x$, defined as the probability of living from age x to $x+1$ given survival to age x. Second, we calculate $\text{EFR}_j(A,t)$ for as the product of $\text{TFR}_j(t)$ for that country and the survival probability until A. In some cases, we will calculate EFR at the region or global level. To aggregate EFR to a set J, e.g., a region, we take the population-weighted average of EFR_j, holding time and age constant. To aggregate EFR across ages, for example the calculation in Eq. (4), we we take a simple average across ages within a country j.[18]

2.2.3 Other demographic concepts

We obtain annual net reproductive rate (NRR) and sex ratio at birth (SRB) for all countries in the period 1950-2019 from the UN World Population Prospects. We calculate replacement level fertility (RLF) for all countries in the period 1950-2019. using the formula we explain in Eq. (15). We do not obtain or use NRR, SRB, and RLF for European data from 1750-1950 or US population by race for 1800-present.

2.3 Connection to the social value of human lives

We justified looking at EFR by saying that the consequences of fertility decline hinge on the number of surviving children, not merely the number of children born. While that may be true, the social value of greater fertility and lower mortality my not be exactly equal to

[16]For European countries before 1950, we do not have TFR data directly from the Human Mortality Database (HMD). Usually, calculation of TFR requires age-specific fertility rates, but HMD does not have that information. Instead, we just use the formula for EFR in Eq. (1), that uses total births, $B(t)$, and the number of reproductive age females, $N_{fR}(t)$. We discuss the difference between TFR and this calculation further in Appendix C.

[17]For Black and White populations in the US, we do not always have life tables that provide survival probability for each 1-year age bin or for each year. Instead, in some cases the life tables are for 5-year age bins (e.g., ages 15-19). In those cases, we assume an equal incremental probability of dying at each age within an age bin. Other times life tables are reported every 10 years, e.g., 1890 and then 1900. In these cases, we assume that annual changes in age-specific survival probabilities are constant between the two reporting dates.

[18]We will sometimes discuss average change in EFR (or TFR or survival rates) across a decade. We obtain this by taking the simple average of annual changes in the relevant demographic statistic.

the number of surviving children. One reason is time discounting. Another is curvature in survival probabilities. A third is that social welfare may not be linear in population. In this section, we set forth sufficient assumptions to justify the use of EFR as a measure of the social value of a larger population.

The social value W of B people born at time t in a given location (e.g., country) is

$$W(B,t) = \sum_{i=1}^{B} \sum_{a=\underline{a}}^{\bar{a}} \beta^a v_i(a, t+a) S_i(a,t) \tag{7}$$

where β is a social discount factor, $v_i(a, t+a)$ is the social value of a person i when she reaches age a at time $t + a$. Here, v reflects both private wage and utility, as well as externalities from an individual, which include gains from trade. Note that v is fairly flexible. It could represent only wages, if only employees earn a surplus in labor markets or it could represent willingness to pay to have a child amortized over a number of years.

We slowly introduce assumptions to simplify the expression for the social value of a population. Let's start with two:

A1. The social discount factor is $\beta = 1$.

A2. Each individual is homogeneous; each individual is at least not so different from the average person in terms of social value that differences can be ignored; or there is a combination of diminishing individual social value and a social insurance scheme such that each person has equal social value.

Under these conditions, the value of the total population in a location is

$$W(B,t) = B \sum_{a=\underline{a}}^{\bar{a}} v(a, t+a) S(a,t) \tag{8}$$

One additional assumption yields an expression very similar to EFR.

A3. Each individual and country uses credit markets to smooth private consumption and external value, respectively, so that social welfare from an individual does not change over the relevant time interval $[t + \underline{a}, t + \bar{a}]$.

This implies that per capita social value is constant over time, i.e., $v(a, t + a) = v$, so that

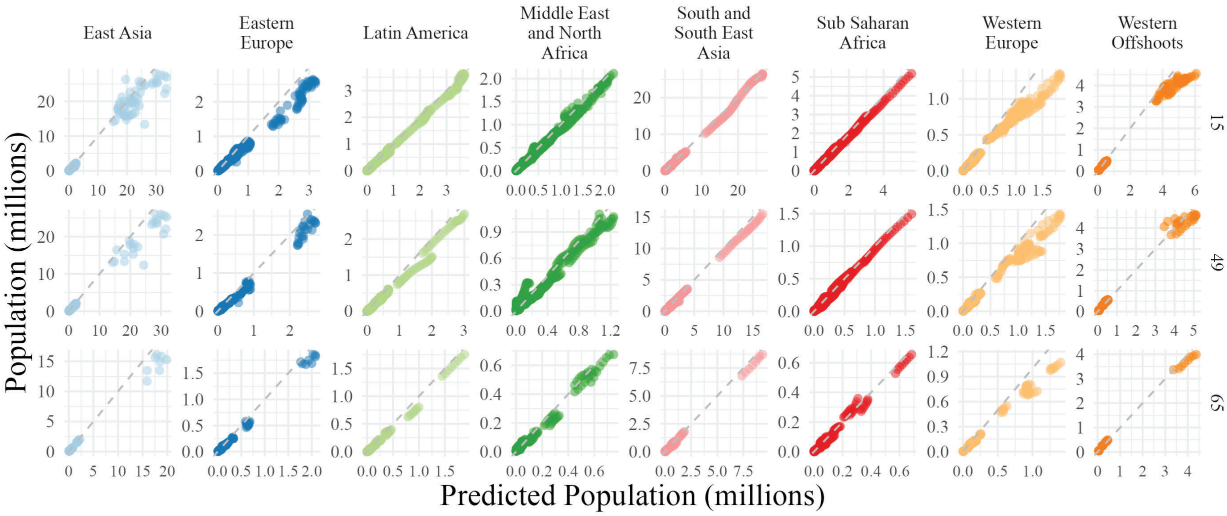
$$W(B,t) = Bv \sum_{a=\underline{a}}^{\bar{a}} S(a,t) \tag{9}$$

If we multiply and divide the right-hand side by N_{fR} (the number of females of reproductive age at time t) and use the fact that $TFR \approx 35 \cdot B/N_{fR}$, we get an expression that includes EFR[19]

$$W(B,t) = N_{fR} \cdot v \cdot \frac{\text{TFR}}{35} \cdot \frac{\bar{a} - \underline{a}}{\bar{a} - \underline{a}} \cdot \sum_{a=\underline{a}}^{\bar{a}} S(a,t) \approx N_{fR} \cdot v \cdot (\bar{a} - \underline{a}) \cdot \frac{\langle \text{EFR(t)} \rangle \underline{a}, \bar{a}}{35} \tag{10}$$

[19]Note that the $\bar{a} - \underline{a}$ and 1/35 do not always cancel because the two terms capture two different concepts: the former is employed to calculate expected EFR, while the latter is used to convert from TFR units (per female of reproductive age over her reproductive lifetime) to per child units. Moreover, B is not explicitly in the right hand side of this equation because it is indirectly an input into EFR.

Figure 1: True age level population is close to predicted age level population.



If we want to convert to units that are comparable to TFR, i.e., social value per female of reproductive age over her reproductive lifetime, we would bring the 35 over to the left-hand side.

To summarize, under assumptions A1 to A3, each child a female has is equal to the product of expected EFR (divided by 35), the number of reproductive age females, the annual per capita social value from a birth, and the number of years that social value is produced. The last of these ($\bar{a} - \underline{a}$) is, e.g., 35 for reproductive EFR and 51 for labor EFR. To economists, the boldest of these assumptions is likely the one about smoothing social value with private and public capital markets.

2.4 Validation with age-specific population

We can validate our measure of EFR by seeing how well it predicts future, age-specific population. To see the theoretical connection between population and EFR, observe that age-specific population is equal to total births a years ago times the probability the people born survived until age a:

$$N_a(t) = B(t-a)S(a, t-a) = \frac{N_{fR}(t-a)}{35}\text{EFR}(a, t-a) \tag{11}$$

Using this, we can show the practical relevance of EFR by showing how well the product of lagged EFR for age a and lagged number of reproductive age females, where the lag is a year, predicts actual age-a population at time t. The results are reported in Figure 1, which plot predicted population (the product of lagged EFR and lagged reproductive age female population) against actual population for different regions (across columns) and different ages (across rows). The correspondence is quite sharp.

2.5 Approximation with existing demographic concepts

It would be convenient if effective fertility could be approximated directly or via simple functions of existing concepts in demography. Here we show that reproductive effective fertility rate (EFR_R) can be approximated by two existing concepts in demography, namely net reproductive rate and the ratio of total fertility rate to the replacement level fertility. However, labor effective fertility rate (EFR_L) does not have readily available approximations.

2.5.1 Net reproductive rate (NRR)

The UN regularly reports the net reproductive rate (NRR), defined as the number of daughters a female will *likely* have over her reproductive years. This measure accounts both for the age-specific fertility of females and the probability that females will survive to each specific age in the reproductive range 15 to 45.

NRR on its face seems like a reasonable approximation for EFR_R, though not for EFR_L. The differences between NRR and EFR_R are minor and/or discretionary, not fundamental. One is that NRR technically takes a weighted average of females survival rates, where the weights are based on age-specific fertility, while EFR_R employs a simple average of female survival rates.[20] A second is that the age range is somewhat different—15 to 45 for NRR and 15 to 49 for EFR. A third difference is that NRR is calculated based on current female (mother's) mortality rates, while EFR_R, as we have defined it, uses future (daughters') mortality rates.[21] These only differ if mortality rates are in substantial flux.

By contrast the difference between NRR and EFR_L is more fundamental, First, the age range for the latter is 15 to 65, which is 20 years further out than NRR and those 20 years (ages 46-65) will have higher mortality rates. Second, NRR employs only female mortality rates, while EFR_L employs male and female rates. Male rates are higher than female rates, as men have lower even age-5 life expectancy than females (Dattani et al., 2023; HMD, 2024).

[20] If $d(a,t)$ is the average number of daughters a women of age a has at time t, then NRR is defined as

$$\text{NRR}(t) = \sum_{a=15}^{49} d(a,t) S_f(a, t-a) = \frac{\sum_{a=15}^{49} d(a,t) S_f(a,t-a)}{\sum_{a=15}^{49} d(a,t)} \sum_{a=15}^{49} d(a,t) \tag{12}$$

Using the fact that the total fertility rate at t ($\text{TFR}(t)$) is approximately the number of daughters per reproductive age female ($d(t)$) times the sex ratio at birth ($\text{SRB}(t)$), i.e., $d(t)(1+\text{SRB}(t)) = \text{TFR}(t)$, we can change the numerator to be children rather than daughters to make NRR comparable to TFR:

$$\text{NRR}(t)\big(1 + \text{SRB}(t)\big) \approx \frac{\sum_{a=15}^{49} d(a,t) S_f(a,t-a)}{\sum_{a=15}^{49} d(a,t)} \text{TFR}(t). \tag{13}$$

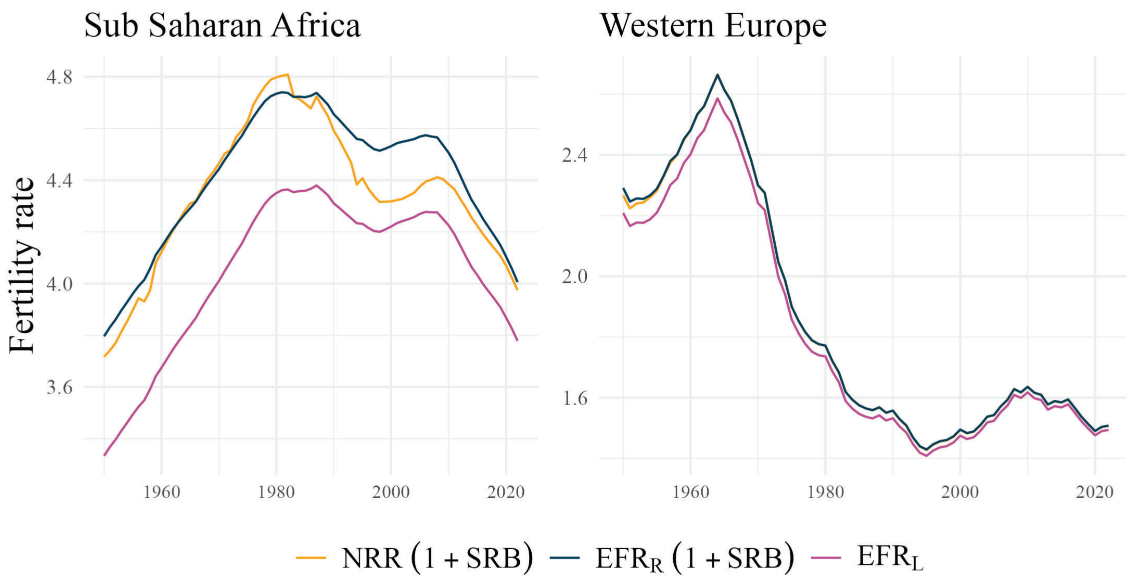
Contrast this with EFR_R:

$$\sum_{a=15}^{49} \frac{\text{EFR}(a,t)}{35} \approx \text{TFR}(t) \frac{\sum_{a=15}^{49} S_f(a,t)}{35} \tag{14}$$

Note that the NRR employs a daughter-weighted average of female survival rates across age while EFR_R employs an unweighted average of female survival across age.

[21] Technically, NRR intends to capture mothers' mortality rates, while EFR_R intends to capture daughters' mortality rates. But this is immaterial, as one could use existing mortality rates to capture daughters' rates and vice versa.

Figure 2: Comparisons of normalized net reproductive rate (NRR) to reproductive effective fertility rate (EFR_R) and to labor effective fertility rate (EFR_L) over time for two regions



Note: EFR is calculated taking average of EFRs across an age range, not EFR at an average age $\tilde{A}$ for a relevant age range. EFR_R and NRR are multiplied by (1+SRB) to convert the number of daughters into the number of children, the units of EFR_L.

On top of these fundamental differences between NRR and EFL_L are two minor, discretionary ones. One is that NRR reports the number of daughters born while EFR_L reports the number of children (male of female) that will survive. This difference can be bridged by multiplying NRR by $1 + \text{SRB}(t)$, where is SRB is the sex ratio at birth at time t. A second discretionary difference is that NRR uses current mortality rates rather than future (projected) rates.

Figure 2 illustrates the utility of NRR as an approximation for EFR_R but not EFR_L. It plots the three measures over time for two regions—Sub Saharan Africa (left panel) and Western Europe (right panel)—which differ dramatically and importantly in mortality profile. The figures normalize NRR and EFR_R so the units are kids per female rather than daughter per female to facilitate comparison to EFR_L. We see that in both regions, NRR is a reasonable approximation to EFR_R. However, because adult mortality rates are relatively high in Sub Saharan Africa, the labor effective fertility rate is lower than both the reproductive effective fertility rate and the net reproductive rate – by roughly one quarter of one surviving child per female for roughly the last 70 years.

2.5.2 Ratio of total fertility rate (TFR) and replacement level fertility (RLF)

TFR is an inadequate measure of EFR because it accounts for fertility, but does not account for survival, as is evident from Eq. (3). In contrast, while replacement level fertility (RLF), defined as the TFR required to maintain a stable (non-changing) population, accounts for mortality, but does not account for actual fertility. When TFR is divided by RLF, however, it can approximate reproductive EFR.

Ignoring migration, RLF is equal to the number of children each mother must have during her reproductive years to produce 1 daughter, divided by the probability that that the daughter will survive through her own reproductive years. The numerator is related to the sex ratio at birth: the number of children required to have a daughter is 1 daughter plus the number of male births per female birth. Thus the formula for RLF (Preston et al., 2000) is:

$$\mathrm{RLF}(t) = \frac{(1+\mathrm{SRB}(t))}{\langle S_f(A,t)\rangle_{15,49}^{d(A)}}, \qquad \langle S_f(A,t)\rangle_{15,49}^{d(A)} = \frac{\int_{15}^{49} d(A)S(A)dA}{\int_{15}^{49} d(A)dA} \tag{15}$$

where $\langle S_f(A,t)\rangle_{15,49}^{d(A)}$ is the weighted average of female survival probabilities from age 15 to 49, where the weights are the number of daughters per age. With a balanced sex ratio of 1.05 and essentially no child mortality, the RLF would be 2.05. Both conditions may not be met in many lower-income countries.

If the weighted average of female survival can reasonably be approximated with a simple average of female survival, i.e., $\langle S_f(A,t)\rangle_{15,49}^{d(A)} \approx \langle S_f(A,t)\rangle_{15,49}$, then we can use Eq. (5) to relate EFR_R to TFR and RLF:

$$\mathrm{EFR}_R(t) \approx \frac{\mathrm{TFR}(t)}{\mathrm{RLF}(t)} \times \frac{2}{1+\mathrm{SRB}(t)} \tag{16}$$

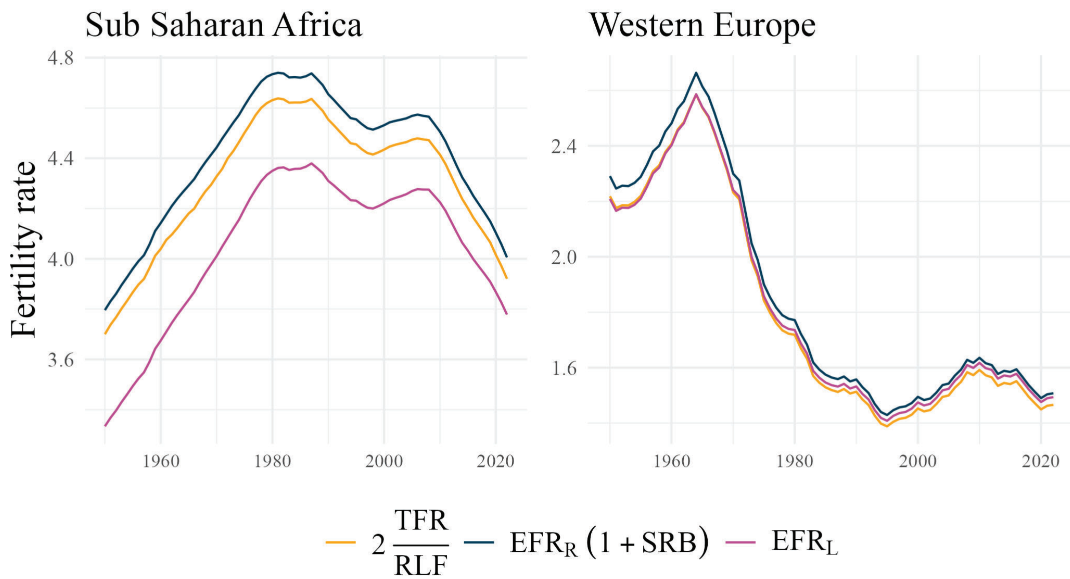
The first term on the right-hand side is the ratio of TFR (accounting for fertility) to RLF (accounting for survival). Because the units of the left-hand side are daughters and the units of the first right-hand side term are children, the second right-hand side term is a conversion factor to compare EFR_R and TFR/RLF. We convert the left hand side to children by multiplying by $1 + \mathrm{SRB}(t)$ to determine the number of children required to get 1 daughter. We multiply TFR/RLF by 2 because it is a ratio of children rather than the number of children. The only modification in the equation above is that we moved the sex-ratio adjustment to the right-hand side.

Like the differences between EFR_R and NRR, the differences between EFR_R and the adjusted ratio of TFR and RLF are minor. However, the differences between the labor EFR_L and the adjusted ratio are larger, and so the latter is a poor approximation for the former, especially in lower-income countries. Figure 3 compares EFR_R, EFR_L and the adjusted ratio of TFR and RLF over time. Like Figure 2, it focuses on two regions, Sub Saharan Africa and Western Europe, that have substantially different mortality profiles. It is evident that the adjust ratio is a reasonable approximation to EFR_R, but not for EFR_L, were mortality rates between ages 50 and 65 are potentially large.

2.6 Comparison to fertility preferences

What is the relationship between fertility preferences elicited via survey and the effective fertility rate? On the one hand, like TFR, desired or ideal fertility may capture only the number of *births* that a parent desires. On the other hand, it may capture the number of *surviving children* that parent desires, which is connected to EFR. Which view is correct

Figure 3: Comparisons of the ratio of total fertility rate (TFR) and replacement level fertility (RLF) to reproductive effective fertility rate (EFR_R) and to labor effective fertility rate (EFR_L) over time for two regions



Note: EFR_R and (TFR/RLF)(2/(1+SRB)) are multiplied by (1+SRB) to convert the number of daughters into the number of children, the units of EFR_L.

depends on how parents interpret a desired fertility survey question, and that interpretation is not a priori obvious. Therefore, we explore empirically the relationship between TFR, EFR and desired fertility. We find that ideal fertility tends to capture desired births, not desired number of surviving children. One implication is that desired fertility is not a reasonable approximation for EFR.

In order to determine whether parents interpret ideal fertility question as a births question or as a surviving-children question depends on how survival influences their answers. If ideal fertility only captures desired births, then ideal fertility—holding desire for surviving children constant—would fall with higher child survival rates. If ideal fertility captures the desired number of surviving children, then idea fertility—holding the desire for surviving children constant—would be unchanged by higher child survival rates. To capture this intuition, imagine a regression of log ideal fertility (IF) on log desired number of surviving children (proxied by EFR) and log expected child survival rate (proxied by projected survival S)[22]:

$$\log \mathrm{IF} = \alpha_0 + \alpha_E \log \mathrm{EFR} + \alpha_S \log S + \epsilon \tag{17}$$

where we have suppressed age and time inputs for exposition and ϵ is the usual regression

[22] Because we are using proxies for a parent's desired number of surviving children and expected survival, we will have measurement error. This measurement error potentially bias coefficient estimates. If we assume classical measurement error and independence of measurement error for fertility and survival measures, then we expect attenuation of coefficients in regression Eq. (18). However, the predicted signs of the coefficients will be preserved despite using proxies.

error term. In the IF-as-birth view, we predict $\alpha_S < 0$ since higher survival should reduce births, holding surviving children constant. In, the IF-as-surviving-children view, we expect $\alpha_S = 0$. Because EFR embeds survival, it may be cleaner to use the definition of EFR (i.e., $\log \text{EFR} = \log \text{TFR} + \log S$) to simplify this regression:

$$\log \text{IF} = \alpha_0 + \alpha_E \log \text{TFR} + (\alpha_S + \alpha_E) \log S + \epsilon \tag{18}$$

If $\alpha_S < 0$ (IF-as-birth view), then the coefficient on log TFR should be greater than that on log survival. If $\alpha_S = 0$ (IF-as-surviving-children view), the coefficients should be the same.

Our analysis supports the view that ideal fertility reflects desired fertility rather than desired number of surviving children. The coefficients on log TFR (reflecting α_E) are all positive (and significant in all but one specification). The coefficient on log survival rate is negative for all specifications, and significant for all specification except ones with two-way fixed-effects. Moreover, in all but one specification, we can reject that the coefficient on log TFR and log survival rate are identical (i.e., that $\alpha_S = 0$). These estimates imply that $\alpha_S < 0$, a result is consistent with the IF-as-fertility view.[23]

Table 2 reports estimates for the regression in Eq. 18 using country-year level data. We obtain TFR and EFR from UN data, as before. We obtain ideal fertility from DHS, which defines it as the number of children that a woman or man would have if they could go back to the time when they did not have any children and could choose exactly the number of children to have in their whole life. Since DHS data are periodic surveys of countries and the date of survey vary across countries, this is neither a large sample nor a balanced sample. On average we have only 2-3 years of data per country and 67-80 countries. The columns vary the source of variation employed for identification using country and year fixed-effect combinations.

[23]There are two major weaknesses in this evidence for the IF-as-fertility view. The coefficients fall in magnitude when country fixed effects are employed, and they become insignificant when two-way fixed effects are included. The first issue is less substantial that the second, as coefficients remain significant with country fixed effects. While the two-way fixed effects estimates use the most credible source of variation, they also have rely on less variation for identification. This required greater power, putting pressure on the small sample size in the DHS data.

Table 2: Relationship between (i) ideal fertility, by sex, and (ii) TFR and survival probability.

	Log Male Ideal Fertility				Log Female Ideal Fertility			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log TFR	0.756***	0.674***	0.208	0.271	0.683***	0.653***	0.257***	0.252*
	(0.171)	(0.164)	(0.128)	(0.169)	(0.106)	(0.098)	(0.072)	(0.126)
Log Survival Rate	-0.793	-1.631**	-0.811**	-0.671	-0.593	-0.996**	-0.561**	-0.388
	(0.716)	(0.775)	(0.298)	(0.486)	(0.403)	(0.383)	(0.248)	(0.274)
Constant	0.294**	0.250*	1.109***	1.040***	0.283***	0.254**	0.888***	0.926***
	(0.141)	(0.138)	(0.151)	(0.275)	(0.096)	(0.097)	(0.069)	(0.186)
Obs.	182	180	159	157	280	279	259	259
No. countries	67	67	44	44	80	79	59	59
Avg. yrs/country	2.7	2.7	3.6	3.6	3.5	3.5	4.4	4.4
R^2	0.671	0.751	0.970	0.979	0.714	0.784	0.953	0.961
TFR = Surv. (p)	0.012	0.001	0.000	0.083	0.000	0.000	0.000	0.047
Country FE	No	Yes	No	Yes	No	Yes	No	Yes
Year FE	No	No	Yes	Yes	No	No	Yes	Yes

Note. Standard errors clustered by country and year in parentheses. * indicates p<0.10, ** p<0.05, *** p<0.01

3 Decompositions

In order to understand the importance and causes of changes in EFR, we take two steps. First, we decompose historical changes in TFR into changes that merely compensate for changes in mortality rates (holding the number of surviving children constant) and changes that alter the number of surviving children that parents have. Second, we drill into changes in EFR and ask how much of the historical change in EFR can be explained by economic factors such as income, prices, structural transformation, and urbanization—four of the most important economic factors affecting the value and cost of surviving children. Our main takeaways are, first, that, since the 1990s, most changes in TFR change the number of surviving children, rather than offset declines in mortality. Second, at most 1/3 of changes in EFR can be explained by our measured economic factors.

3.1 Decomposing TFR into EFR-compensatory and EFR-altering buckets

To start, we decompose changes in TFR into changes that hold EFR constant (EFR-compensatory changes) and changes that alter EFR (EFR-altering changes). Our units will be percentages, to simplify both derivation and interpretation.

The percentage change in TFR at time t can be written as:

$$\frac{\partial \log \mathrm{TFR}(t)}{\partial t} = \frac{\partial \log \mathrm{TFR}(t)}{\partial t}\bigg|_{\frac{\partial \langle \mathrm{EFR}(t) \rangle}{\partial t}=0} + \left[\frac{\partial \log \mathrm{TFR}(t)}{\partial t} - \frac{\partial \log \mathrm{TFR}(t)}{\partial t}\bigg|_{\frac{\partial \langle \mathrm{EFR}(t) \rangle}{\partial t}=0}\right] \tag{19}$$

where we take a simple average of EFR over ages 15 to 65.[24] The first term on the right hand side is the EFR-compensatory change in TFR. The second term is the EFR-altering change in TFR.

We can simplify both types of changes using the fact that $\langle \mathrm{EFR}(A,t) \rangle = \mathrm{TFR}(t) \cdot \langle S(A,t) \rangle$. This fact implies that a compensatory change in TFR that holds EFR constant at time t is

$$\frac{\partial \log \mathrm{TFR}(t)}{\partial t}\bigg|_{\frac{\partial \langle \mathrm{EFR}(t) \rangle}{\partial t}=0} = -\frac{\partial \log \langle S(A,t) \rangle}{\partial t} \tag{20}$$

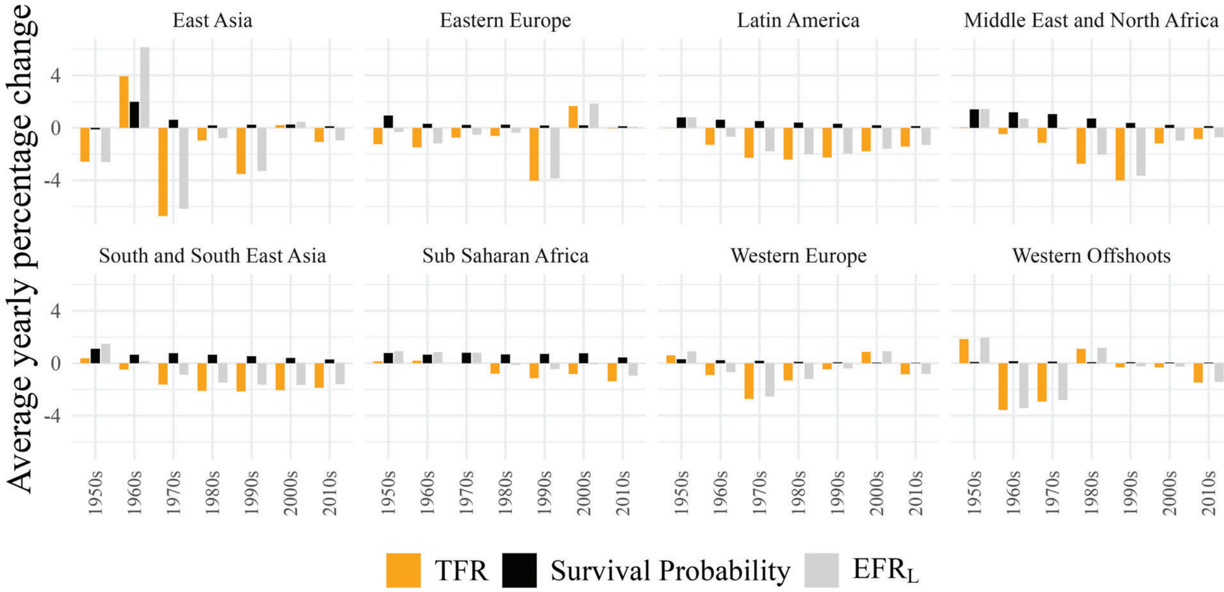
Plugging this into the previous equation, we see that the first term is the inverse of the percentage change in survival rates and the second term is just the percentage change in EFR:

$$\frac{\partial \log TFR(t)}{\partial t} = -\frac{\partial \log \langle S(A,t) \rangle}{\partial t} + \frac{\partial \log \langle EFR(A,t) \rangle}{\partial t} \tag{21}$$

In this section, we apply this decomposition using country-year data from the UN from 1950-2019 and taking averages over the 15-65 age range for both sexes. Figure 4 shows the

[24] One could also do this exercise using female survival and ages 15 to 49, to connect changes in TFR to changes in reproductive EFR. In this section we will focus on labor EFR and drop references to age subscripts (15,65) in averages to simplify notation.

Figure 4: Percent changes of TFR, Survival Probability and EFR_L by region and decade.



Note: Regional statistics reflect population-weighted average across countries in a region. Percent change in TFR equals percent change in EFR_L minus percent change in survival probability.

region-level average percent changes in TFR by decade, as well as its components percent change in survival probability and percent change in EFR.

Our analysis yields four conclusions. First, for regions that have had high survival probabilities throughout the 70-year period (such as Western Europe and the Western Offshoots), most changes in TFR have been EFR-altering changes. Second, for many regions with higher mortality rates (such as South and South East Asia, the Middle East and North Africa, and Latin America), there has been a shift over time. For the first few decades in our time series, changes in TFR were more EFR-compensatory changes. However, at around the half way point in the time series, we see a shift where EFR-altering changes start to dominate. This is due not just to slowing of declines in mortality rates, but also a sustained decrease in TFR.

East Asia and Sub Saharan Africa are outliers. The former look like Western Europe and offshoots with the exception of the 1960s. East Asia is dominated (in terms of population) by China, and China experienced the Great Famine between Spring 1950 and end of 1961. During the famine, birth rates plummeted and mortality spiked. When the famine ended, fertility spiked and mortality fell. Thus, when TFR rose after the famine, it actually has larger, positive effects on EFR because mortality also fell. Sub Saharan Africa is unique for a different reason: mortality started higher and the declines in mortality persist throughout the 70 year period of our data. Moreover, TFR rises until the 1980s. Thus, EFR rises in the first 3 decades of our sample, and then declines, but by less than the change in TFR.

While Figure 4 decomposes percentage change in TFR, one might also want to understand the relative role of mortality rate changes and EFR changes as a *share* of percentage change in TFR. To obtain the share of change in TFR that can be attributed to EFR-compensatory and EFR-altering forces we switch to discrete time and normalize Eq. 21 by the percentage change in TFR:

$$1 \approx -\frac{\Delta\langle S_t(A)\rangle}{\langle S_t(A)\rangle}\frac{TFR_t}{\Delta TFR_t} + \frac{\Delta\langle EFR_t(A)\rangle}{\langle EFR_t(A)\rangle}\frac{TFR_t}{\Delta TFR_t} \tag{22}$$

where $x_t(z)$ is the discrete time analogue to $x(z,t)$ and $\Delta x_t = x_t - x_{t-1}$.[25] Table 3 calculates the relative role of EFR-compensation and EFR-alteration components of fertility change in each region since 1950.

Across the whole world, the EFR-altering component of fertility change is roughly twice as large as the EFR-compensating component. With the exception of the Middle East and North Africa, South and South East Asia, and Sub Saharan Africa, fertility changes alter EFR much more than they merely compensate for decline in mortality rates. However, in the Middle East and North Africa and in South and Southeast Asia, the two components play roughly equal roles. And in Sub Saharan Africa, TFR declines are insufficient to compensate for mortality declines, so EFR actually rose.

Table 3: Share of change in TFR attributable to EFR-compensatory changes and EFR-altering changes, on average from 1950-2019 by region and for the whole world.

	1950-2019	
Region	Compensatory	Altering
East Asia	0.31	0.67
Eastern Europe	0.34	0.67
Latin America	0.26	0.74
Middle East and North Africa	0.49	0.51
South and South East Asia	0.45	0.56
Sub Saharan Africa	1.30	-0.29
Western Europe	0.20	0.80
Western Offshoots	0.11	0.89
World	0.36	0.63

[25] Our conversion to discrete time means that the sum of the two terms in Eq. (22) will be only approximately equal to 1. When we take the percent change in EFR we get

$$\%\Delta\langle EFR_t(A)\rangle = \frac{\Delta TFR_t\langle S_t(A)\rangle}{TFR_{t-1}\langle S_{t-1}(A)\rangle} + \frac{TFR_{t-1}\Delta\langle S_t(A)\rangle}{TFR_{t-1}\langle S_{t-1}(A)\rangle} \tag{23}$$

$$\%\Delta\langle EFR_t(A)\rangle = \%\Delta\langle TFR_t(A)\rangle\big(1 + \%\Delta\langle S_t(A)\rangle\big) + \%\Delta\langle S_t(A)\rangle \tag{24}$$

where, in the limit as time increments converge to 0, $1 + \%\Delta\langle S_t(A)\rangle$ will be 1. This means that for decades when a region experienced large changes in survival probability, the two terms will be somewhat different than 1. A value below 1 indicates large decrease in survival probability where as a value greater than 1 indicates a large increase in survival probability.

3.2 Historical Data on Europe and on American Sub-populations

Our country-year data begin in 1950. However, our decomposition can be used to understand earlier changes in TFR. In this section we examine two data sets that focus mainly on the period from 1800-1950, the early years of the Industrial Revolution. Our highest-level takeaway are that fertility changes during this period in now-developed countries look a lot like fertility changes in developing countries in the period from 1950-1990, i.e., they substantially compensate for changes in mortality.

3.2.1 Historical European Data

Data on fertility rates during the 19th century are available for a dozen European countries. For most, data only start in 1850. But for Sweden and France, data start in 1750 and 1825, respectively. The data are from the Human Mortality Database. We complement those with post 1950 data from the UN. The Human Mortality Database data do not include age-specific fertility rates so we cannot calculate TFR by summing up age-specific birth rates (Preston et al., 2000). Instead, we approximate TFR using total births in each year, which means different ages are not weighted equally. However, we show in Appendix C that this TFR approximation tracks TFR in the post-1950 UN data reasonably well. To ensure that the pre- and post-1950 data are comparable, we calculate approximate TFR for the post-1950 data from the UN since that is the only statistic we can calculate for the pre-1950 data. We calculate survival probability as the average survival probability of ages 15 to 65, as described in Eq. 4.

We report our analysis of this approximated TFR across two figures. Figure 5 plots approximated TFR (yellow), projected survival rates (black), and EFR_L (purple) over time. Figure 6 decomposes approximate TFR into changes that compensate for changes in survival probability and those that change EFR_L for five select countries from different regions of Europe, and for all available European countries.

We draw three conclusions from Figure 5. First, EFRs are remarkably low and stable even during demographic transitions (marked by vertical dotted lines).[26] During those transitions, EFRs stay below 3 in all countries other than Iceland, even as TFR falls from levels as high as 5 to around 2. An implication is that early changes in TFR mainly compensated for a decline in mortality rates. And those TFR adjustments were effective at controlling EFR even though, according to demographic transition theory, they followed mortality declines with a lag.

Second, for the few countries for which we have pre-transition data (Denmark, England, Netherlands, Norway, Sweden), we see that the demographic transition as often reduced effective fertility as it increased. We suspect the reason is that mortality declines that triggered the demographic transitions made people born before the transition live longer.

Third, while EFRs fall below two after demographic transitions, they often remain stable at that low level. For example, Belgium, Denmark, England, Netherlands, and Switzerland have EFRs below 2 after roughly 1975, but EFRs have remained flat since then. This suggests that EFRs are below replacement, but do not always continue to decline.

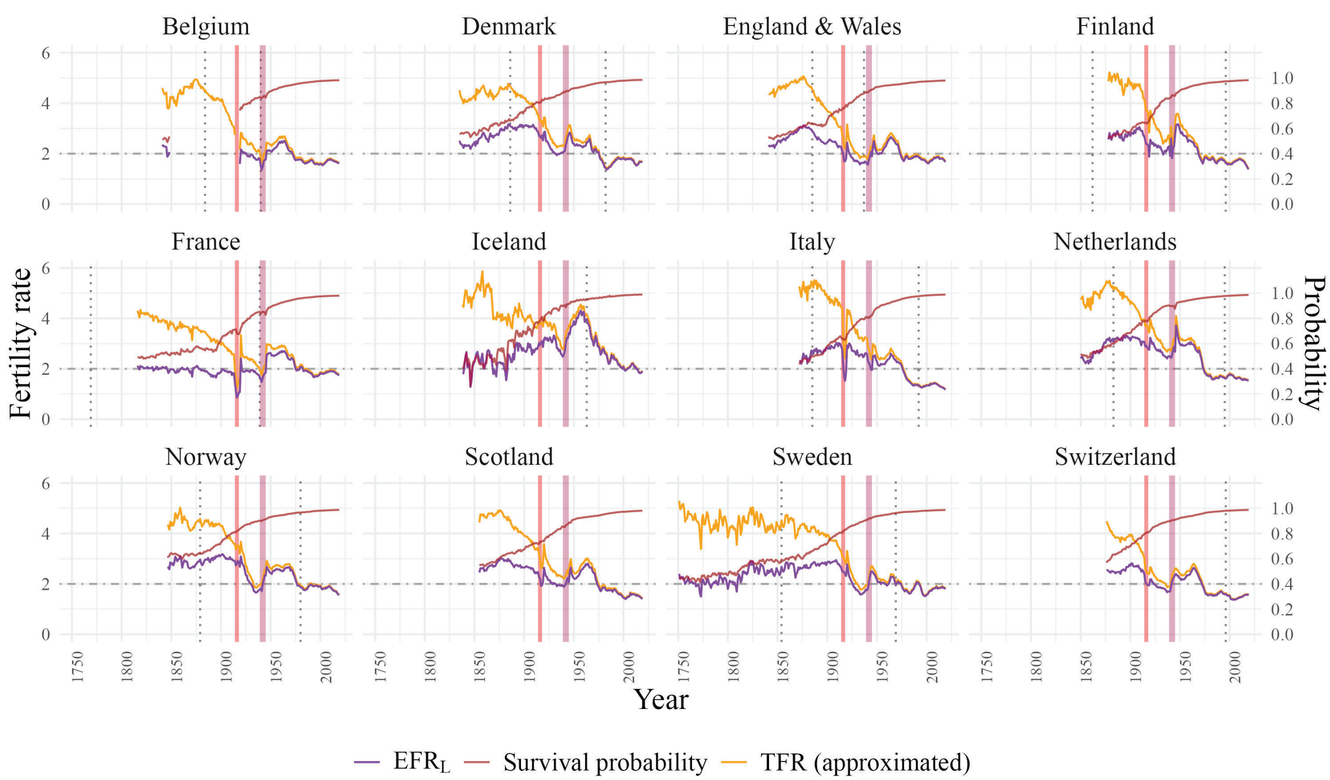
[26] We obtain start and end dates for demographic transitions from (Delventhal et al., 2021).

Aside from evidence that early TFR changes compensate for mortality declines during the demographic transition, Figure 6 reveals several interesting facts about TFR and EFR in Europe over the 20th and early 21st centuries. First and foremost, TFRs (orange bars) and EFRs (grey bars) fluctuate quite a bit, both upwards and downwards, during this period. The cycling in TFR and EFR since 1900 is consistent with Easterlin's hypothesis that fertility cycles in response to lagged effects of fertility on labor markets (Easterlin, 1976).[27] Of course, our data are only capable of showing changing trends, not the causes of those trends.

Second, the spikes and dips coincide across countries, and also with world events that might have altered peoples' value of sacrificing their own consumption to have children. For example, the 1920s show a decline in TFR and EFR almost everywhere. This is consistent with the view that, post-World War One and the 1918 flu pandemic, people had high discount rates and valued present-consumption over the future, perhaps because events in the teens reminded people that life was short (Van Bavel, 2007). Many places—though not all places—experienced a post-war spike in fertility and EFR. There is a literature on possible causes (e.g., Van Bavel and Reher, 2013), though among them is the view that the end of World War Two led to an era of optimism that coincided with higher fertility rates. Fertility fell sharply in the 1970s. This is consistent with rising concerns about over population or an increase in the returns to college during this period (De Silva and Tenreyro, 2017, 2020). Finally, a few places saw a drop in fertility and EFR in the 2010's. It may be too early to tell why this occurred, but some have speculated about heightened pessimism about the future due to climate change (Blum, 2020).

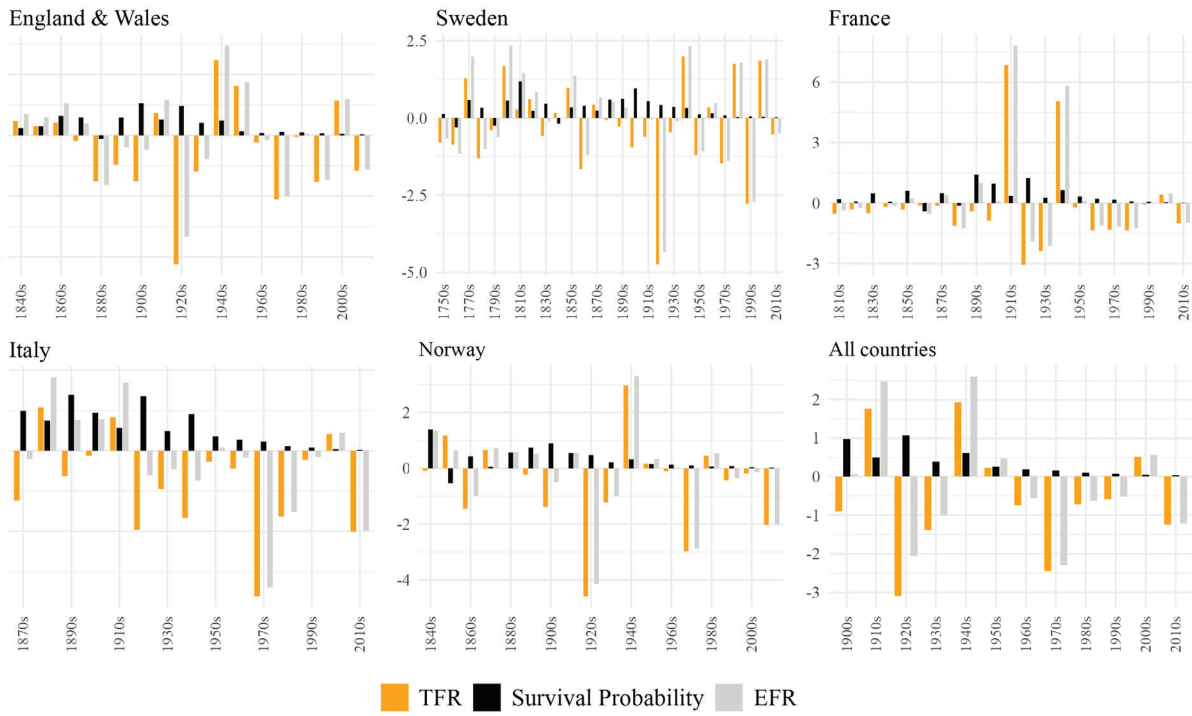
[27] Easterlin explains fertility cycles through the interaction of economic conditions and relative cohort size. Larger birth cohorts face tougher job markets and lower relative income, leading to delayed marriage, fewer children, and smaller birth cohorts. Conversely, smaller cohorts experience better economic opportunities, encouraging earlier marriage, higher fertility and larger birth cohorts. Together these create cyclical patterns over generations.

Figure 5: EFR_L is more stable than TFR.



Note: The x-axis shows time, and the y-axis shows approximate TFR (yellow) and EFR (purple) in units of children per female over her reproductive life. Each subplot is a different country. The start dates for different countries varies in accordance with data available from the Human Mortality Database.

Figure 6: Percent changes of TFR, survival probability, and EFR_L by decade for five select countries and all countries.



Note: Countries in the "All countries" plot are Denmark, England and Wales, Finland, France, Iceland, Italy, Netherlands, Norway, Scotland, Sweden, and Switzerland.

3.2.2 US Black vs. White populations

We obtain historic data on US populations by race from multiple sources listed in Table 1. The data start in 1800 for White populations and 1850 for Black populations. Unlike with historic European data, we have access to TFRs. However, we do not have access to mortality rate projections. Therefore, we use current-year survival rates to compute EFRs. Appendix B shows that this approximation is not unreasonable.

While Black TFR is much higher than White TFR over time, there is—remarkably—almost no difference in number of surviving children across races. Figure 7a plots TFR and EFR separately for each subpopulation. Figure 7b plots explicitly the differences in TFR and in EFR to highlight how different TFRs are and how similar EFRs are across races. On the one hand, these figures indirectly highlight the massively higher mortality rates among Black populations. That is the only variable that explains why TFRs can be different but EFRs are nearly identical across races. This gap in mortality is well documented in the literature (e.g., Chay and Greenstone, 2000). On the other hand, the figure shows how similar choices about number of surviving children are across the races. This finding, as far as we know, is novel. Candidly, it is also a departure from our priors.

Decomposing changes in fertility by race explicitly into EFR-compensating (survival probability) and EFR-altering components (Figure 8) highlights four results. First, TFR

Figure 7: Comparison of US Black and White population fertility metrics over time since 1850 and 1800, respectively.

(a) Black and White TFR and EFR_L (b) Difference in EFR and in TFR across race

Note: These calculations use current mortality rates, not projections.

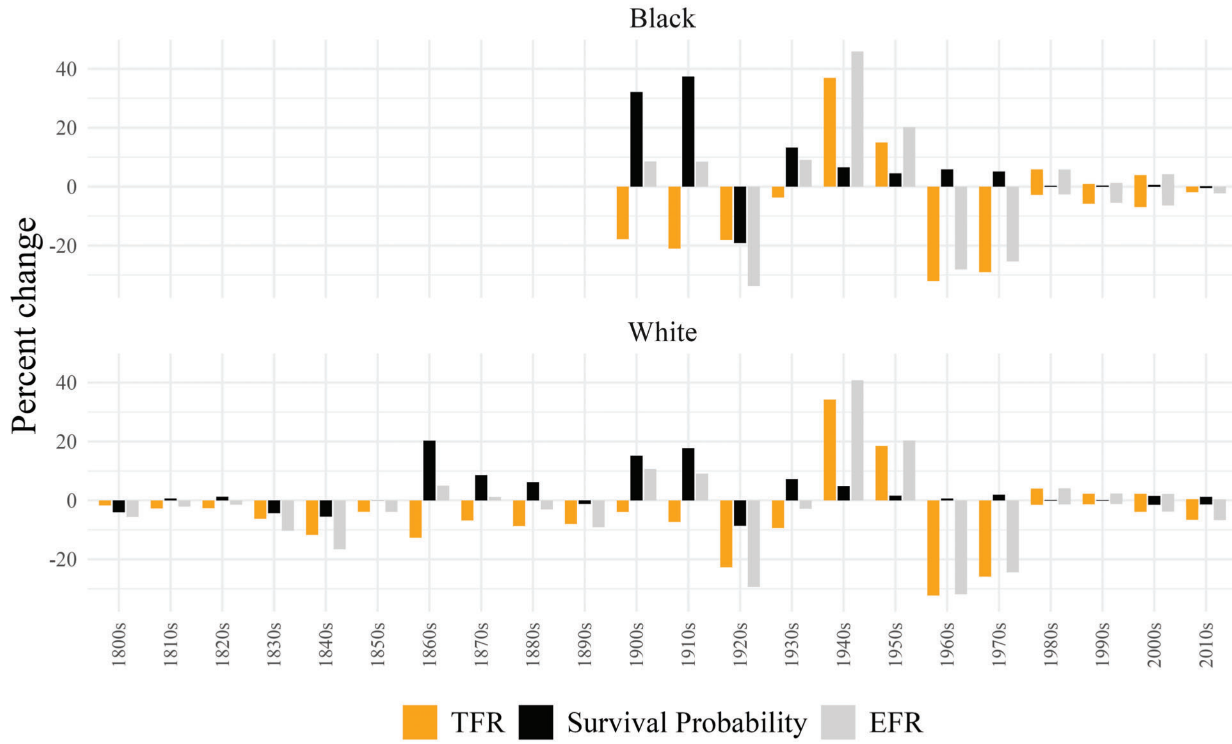
changes are small before 1850, when the Second Industrial Revolution (or the American Industrial Revolution) begins (Mokyr and Strotz, 1998). We glean this from data on White populations, which are provide the only data starting in 1800. (Black mortality rates are only available starting in 1900.) Second, mortality mainly declines and TFR mainly addresses those declines in the period from 1850-1950. This is the period that covers substantial changes in urbanization in America (Boustan et al., 2018). After this period, TFR changes mainly change the number of surviving children parents have. Third, there are large fluctuations in TFR in the 20th century for both races. This contrasts with the stead and small declined in TFR among White populations in the 19th century. Finally, there a remarkable coincidence of TFR and EFR fluctuations amongst both Black and White populations in the 20th century. This suggests common shocks across those two populations in the US.

3.3 Decomposing EFR into economic and non-economic correlates

Having explored the extent to which changes in fertility can be decomposed into changes that compensate for mortality changes and changes that alter the number of surviving children or EFR, we next examine the economic correlates of EFR. We do this in three ways. First, we explore the slope of the relationship between EFR and income. Second, show the distribution of EFR conditional on income. Finally, we estimate the fraction of variation in EFR that can be explained by income and a host of other economic variables.

Like TFR, EFR is negatively correlated with income. Figure 9 uses data from 165 countries and 4 years (1950, 1975, 2000 and 2019) to plot smoothed values from a population-weighted local polynomial regression of fertility (either TFR or EFR) on PPP-adjusted per capita GDP. Each regression uses data from a single year. The left subplot uses TFR and the right plot uses EFR to measure fertility. We draw two conclusions from this Figure. First, the TFR-income gradient does not appear to shift very much over time. It is true that the 1975 gradient (red line) is above the 2000 and 2019 gradients, but it is also above the 1950 gradient. Second, the major difference between the TFR and EFR plots is that the income

Figure 8: Decomposition of Black and White TFR into EFR compensating and EFR altering components



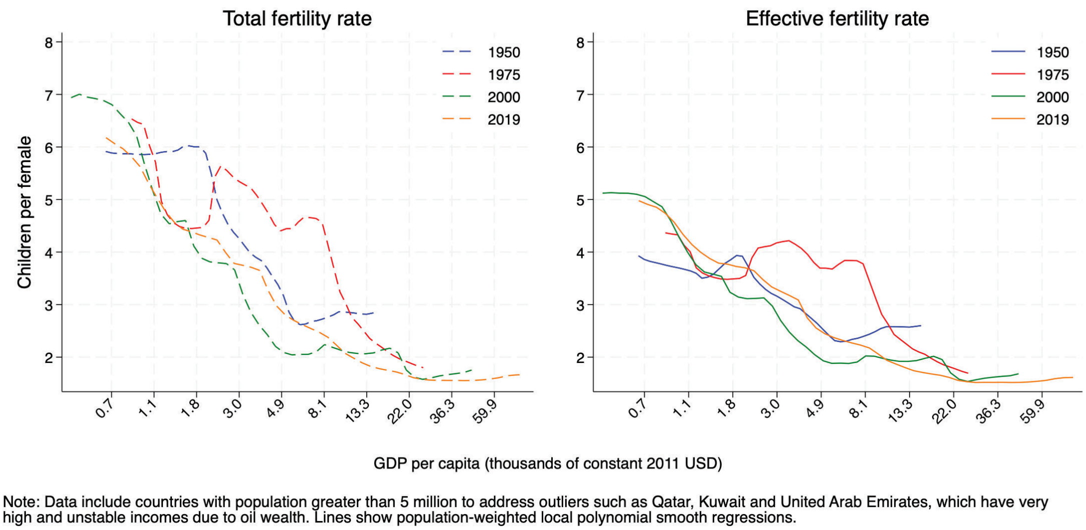
Note: These calculations use current mortality rates, not projections.

gradient is flatter for EFR, suggesting some of the TFR-income slope captures the negative relationship between mortality rates and income.

The relatively consistent relationship between fertility and income masks substantial variation in fertility conditional on income. Figure 10 plots the population-weighted distribution of fertility in either 1975, 2000, or 2019 across countries with income in the range $[\$5835.20, \$8752.80]$. These bounds are equal to 80% to 120%, respectively, of the median income of all countries using data from all three years (1975, 2000, and 2019).[28] The distributions of TFR and EFR are plotted with dashed and solid lines, respectively. Our primary take-away is that there is substantial variation fertility (EFR or TFR) among middle income countries even in 2019. Second, fertility—however measured—has both declined and converged over time: both the mode and variation in fertility across countries were much larger in 1950. Third, although the mode of the EFR distribution is lower than that of TFR, it does not appear that the variation in EFR differs much from variation in TFR. Since variation in EFR is roughly proportional to the product the variance of TFR and mortality rates, this suggests low variation in mortality rates conditional on income. That said, the decline in mortality rates over time means that the distributions of TFR and EFR have converged to each other.

[28]This range of income corresponds to lower-middle income countries under World Bank definitions. Our results do not change if we narrow the range of income to 90-110% of the median $7,294.

Figure 9: Relationship between fertility and PPP-adjust per capita GDP in 1950, 1975, 2000, and 2019.

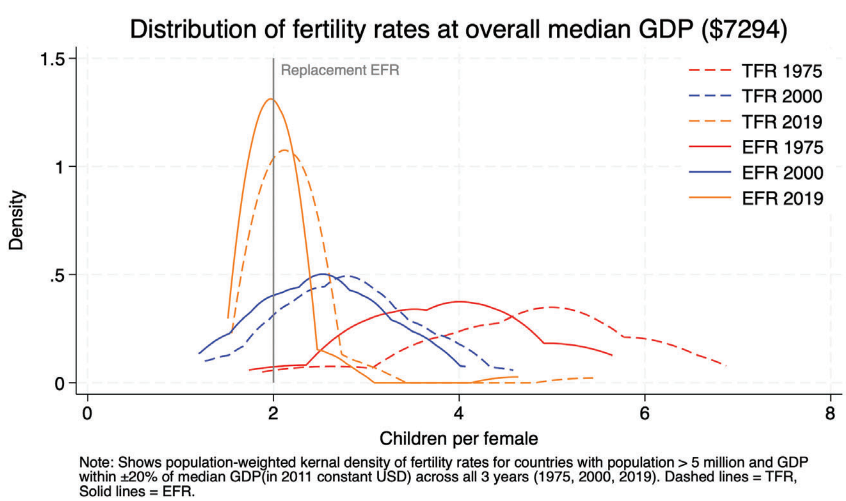


To quantify the extent to which the overall variation in EFR can be explained by income—as well as other notable economic factors—we employ estimated R^2's from simple regression models of EFR on income and these other factors. Specifically, we estimate variations of the following population-weighted regression:

$$y_{jt} = \beta \mathbf{Z}_{jt} + \gamma_j + \delta_t + u_{jt} \tag{25}$$

where j indexes countries, y_{jt} a measure of fertility, and γ_j and δ_t are country and year fixed effects. Our primary measure of fertility is the effective fertility rate EFR_{jt}, but we will also look at TFR_{jt} for comparison. The vector $\mathbf{Z}_{jt}$ are economic factors. All other factors are

Figure 10: Distribution of fertility conditional on income in 1975, 2000, and 2019



captured by fixed effects and the residual.

We define economic factors to be income, prices of goods, farm share of employment, urban share of population, and returns to education. In both economic theory and empirical literature, these are the most important drivers of equilibrium levels of fertility. Income is thought, in part, to increase demand for surviving children because children are "normal goods". However, it can also push down demand for such children either because it increases demand for quality rather than quantity of kids or because it increases the time cost of raising children. Price levels increase the cost of surviving children because raising children requires market purchased goods and services, and the cost of those are included in price indices. We measure the combination of income and prices with PPP-adjusted log GDP per capita. Farm share of employment increases demand for children because farms are often owned by families and farmer often have children to work on their family farms. Urban share of the population measures the cost of children because housing costs are higher in urban areas and housing space is an important cost of raising children. Finally, we use educational attainment data from Barro and Lee (2013) to estimate the returns to education and the demand for quality of children, which due to the budget constraints, will ceteris paribus reduce the quantity of children. (Data sources are further discussed in Section 2.2.1.) To be fair, there may be other economic factors that play a role in the demand and supply of children. However, the ones we use are surely the most influential. Indeed, as we shall see, PPP-adjusted income explains up to 60%[29] of the variation in EFR that economic factors explain (.

There are multiple ways to measure the importance of economic factors for determining effective or total fertility. One way is to estimate Eq. (25) without fixed effects and calculate the R^2. This is an upper bound on the influence of the economic factors we include because it attributes to economic factors any variation in fixed effects (i.e., time- or geography-invariant factors) that can be explained by economic factors. Some of that variation in fixed effects that is correlated with economic factors may actually be "caused" by fixed effects which also drive economic factors rather than be caused by economic factors which drive time- or geography-invariant means.[30]

Another way to measure the influence of economic factors is to regress EFR on two-way fixed effects and regress the resulting residuals on economic factors. The product of $1 - R^2$ from the first regression and R^2 from the second regression is a lower bound on the influence of economic factors under the same logic for why omitting fixed effects from Eq. (25) yields an upper bound on the role of economic factors. This lower-bound approach attributes all influence of economic factors that are correlated with fixed effects to those fixed effects.[31]

[29] In Table 4, $60\% \approx 0.365/0.604$. The fraction is lower (1/9) at the lower bound.

[30] More precisely, any variation in surviving children that should be attributed to time-invariant or location-invariant factors are credited to our economic factors due to omitted variable bias when non-economic factors are excluded from our regression, to the extent the non-economic factors are correlated with economic factors. This is a weak upper-bound because non-economic factors can only have non-negative explanatory power.

[31] There are other perhaps more constraining bounds on the influence of economic factors. For example, one might employ the partial R^2 of economic factors as a lower bound for their influence. This statistic would calculate the proportion of variance in EFR that can be explained by economic factors after removing variance in EFR and economic factors that can be explained by fixed effects. The problem is that this reduces the variance of the dependent variable EFR, along with that of the economic factors. So it is not an apples-to-apple comparison. Alternatively, one might employ the semi-partial R^2 of economic factors as

Table 4: Extent of variation in EFR or TFR that can be explained by economic factors, including income.

	Upper bound	Intermediate (Year FE)	Intermediate (Country FE)	Lower bound
Effective Fertility Rate (EFR)				
Four economic factors	0.604	0.397	0.095	0.018
Log GDP per capita only	0.365	0.235	0.030	0.002
Total Fertility Rate (TFR)				
Four economic factors	0.737	0.464	0.088	0.005
Log GDP per capita only	0.550	0.347	0.034	0.000

Note: Table reports the fraction of variance in EFR or TFR attributable to four economic factors (PPP-adjusted GDP per capita, agriculture share of employment, urban share of population, average years of schooling) and to PPP-adjusted GDP per capita alone. Data are from up to 165 countries from 1950-2019 – though this is not a balanced sample. The upper bound is the R^2 from a regression of EFR on the 4 factors. The lower bound is the product of $(1 - R^2)$ from a regression of EFR on country and year fixed effects and the R^2 from a regression of the resulting residuals on the 4 economic factors. We interpret this as the influence of economic factors using only variation within country and across years. The middle bounds are the product of $(1 - R^2)$ from a regression of EFR on either year or country fixed effects (as indicated in the column header) and the R^2 from a regression of the resulting residuals on the 4 economic factors. We interpret these as the influence of economic factors using only cross-sectional or time-series variation, respectively.

A third measure gives an intermediate estimate of the influence of economic factors. In this approach, we modify the approach that yields a lower bound by regressing EFR on either (i) only year fixed effects (to remove the influence of, e.g., differences in fertility over time) or (ii) only country fixed effects (to remove the influence of, e.g., differences in fertility across countries at the start of our sample in 1950). We then regress the residuals against economic factors. Now the product of $1-R^2$ from the first regression and R^2 from the second regression yields an intermediate estimate of the influence of economic factors.

Our lower bound estimates suggest our 4 economic factors explain just 1.8% of variation in EFR if the source of identifying information is variation within country and across years (top panel Table 4). Our upper bound indicates that economic factors can explain potentially 60.4%, and log GDP per capita can explain almost half or 36.5%, of variation in EFR. This is a wide range. But it suggests that 40% or more of variation is *unexplained* by our four economic factors. Our conclusion is that non-economic factors, such as culture, may play an important role in global EFR changes.

Our intermediate estimates of influence of economic factors suggest that economic factors explain the cross-sectional variation more than the time-series variation in EFR across the world. When we control for the time series variation with year fixed effects, economic factors still explain nearly 40% of the variation in EFR. But when we control for the cross-sectional variation, the value added by economic factors falls to less than 10%.

an upper bound. This statistic would calculate the proportion of variance in EFR that can be explained by economic factors after removing variance (only) in economic factors that can be explained by fixed effects. The problem with this measure is that it is still attributing variance in fixed effect means that are “caused” by economic factors to non-economic factors. I.e., it still underestimates the possibly true role of economic factors.

Finally, the bottom panel of Table 4 reports how much of the variation in TFR can be explained economic factors. Apparently, economic factors explain more of the variation in TFR than that in EFR. Intuitively, this is because economic factors also explain differences in survival rates. Where as EFR nets out mortality rates from TFR, TFR captures the influence of both mortality rates and EFR.

One might worry that our analysis underestimates the role that economic factors play in explaining EFR because we do not capture the universe of economic factors that might affect EFR. That concern is certainly warranted. But its import may be limited. It seems reasonable to believe that the factors we considered includes five of the most important economic variables as measured by explanatory power. Other economic factors that we omit likely explain less.

4 Conclusion

In this paper we offer a new measure of the number of surviving children per reproductive age female. It is motivated by economic models, which posit that households care about the number of children who survive—not merely the number who are born. It can also be used to test those models. Our measure of surviving children can be specialized—via EFR_L and EFR_R—to measure growth in the population that produces or that reproduces, respectively.

Our measure has several advantages. First, it is simpler to measure replacement level effective fertility. Replacement EFR_L is 2 across countries and time, regardless of mortality. Replacement EFR_R is roughly 2 with balanced sex ratios, whereas replacement level fertility (RLF) varies with mortality even with balanced sex ratios. Second, reproductive EFR can be approximated with existing demographic concepts, specifically NRR and the ratio of TFR to RLF.

The main drawbacks of our measure are that there is no reasonable approximation for labor EFR and EFR can be data-intensive to calculate. However, we show that there are simple approximations for both EFR_L and EFR_R that use single ages rather than averages over ages and that use current mortality rates rather than mortality projections. In addition, we will make our measures of EFR publicly available.

We explore trends in EFR using data on 1965 countries between 1950-2019, historical data on European countries, and historical data on racial subpopulations in the US. Across the world, we find that mortality declines explain roughly one-third the decline in TFR since 1950. Declines in EFR explain the rest. Moreover, economic factors such as income explain at most 60% of the variation in EFR during this period, suggesting a potentially influence of non-economic factors such as culture on the number of surviving children that households produce. Across Europe we find remarkable stability in EFR before and after 1910-1975, a period spanning the two World Wars. During this interregnum, we see substantial cycling of EFR, a topic for future research. We also find remarkable concordance of EFRs across Black and White populations in the US, even though the two groups have dramatically different mortality profiles.

Our work suggests a number of natural avenues for future work. In some sense our measure of EFR is incomplete because it assumes that all children will work or all daughters will reproduce. The rise in the out-of-labor force population and decline in fertility suggests this assumption is incorrect. A task that remains is to convert EFRs into a measure of social assets by deflating them using projected employment or fertility rates. Another, related avenue for work is to examine trends in total human capital production over time. This could be captured by with the product of EFR and measures of health or education.

Finally, our measures of EFR can be used to test quality-quantity models or models of cultural change. One example is to see if shocks to EFR change educational investments in surviving children, in the spirit of Doepke (2005). Another is to examine whether there is social contagion in EFR, in the spirit of Delventhal et al. (2021).

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Online Appendix

"Fertility versus surviving children"

Anup Malani, Ari Jacob

APPENDIX

In this appendix we explore approximations to effective fertility rate (EFR) that require less data or calculation than EFR as defined in the text.

A Single-year EFR approximation for average EFR

In the main text, we calculate average EFR over a range of ages (e.g., reproductive period or working period) to obtain an EFR that captures the value of living even a portion of a reproductive or labor life span. However, it might be preferable to calculate EFR at a single age in the range rather than an average EFR in the range to reduce data and calculation requirements.

One candidate for a single age is *average age* in a range. This choice of age would allow us to compare approximate EFR to other demographic concepts, such as net reproductive rate (NRR), that are not an average across ages, but are calculated at particular age, e.g., the mean in a range. A downside of this choice is that, because calculating average age in a range requires survival probabilities for each age, calculating EFR at average age may not reduce data requirements relative to average of EFR across ages. Another candidate for choice of age is the *midpoint age* in a range. This choice is definitely requires less data. However, the corresponding approximate EFR is somewhat less comparable to, e.g., NRR.

In this section we examine each single-age approximation in turn.

A.1 Average age

To justify our approximation of average EFR across a range of ages with EFR at the average age in that range, we take advantage of the fact that survival rates are roughly linear in the relevant age range, even across very heterogeneous regions of the globe (Preston et al., 2000). For average age of a worker, for example, we calculate this average age as the average age of a person who survives past the first age at which a person works (15), up to a maximum of 65. Formally, if an individual works from ages $\underline{a}$ until $\bar{a}$ and $p(A,t) = S(A,t) - S(A+1,t)$ is the probability of living to exactly age A given birth in year t, then the relevant age for which to calculate EFR is

$$\tilde{A}(\underline{a}, \bar{a}, t) = \min\left\{\frac{\sum_{A=\underline{a}}^{\bar{a}} p(A,t)A}{\sum_{A=\underline{a}}^{\bar{a}} p(A,t)}, \bar{a}\right\} \tag{26}$$

(Because survival rates vary across location and time, $\tilde{A}$ will also vary across location and time.) Our EFR approximation then calculates EFR of the resulting average age.

One can use this single-year EFR approximation for average EFR when calculating reproductive and labor EFR:

$$\text{EFR}_R(t) \approx \frac{\text{TFR}(t)}{1+SRB(t)} S_f(\tilde{A}_f(15,49,t),t) = \text{EFR}_f(\tilde{A}_f(15,49,t),t)$$
$$\text{EFR}_L(t) \approx \text{TFR}(t) S(\tilde{A}(15,65,t),t) = \text{EFR}_L(\tilde{A}(15,65,t),t)$$

where $\tilde{A}_f(15,49,t)$ is the average age of reproductive age females and $\tilde{A}(15,65,t),t)$ is the average age of working-age people (males and females).

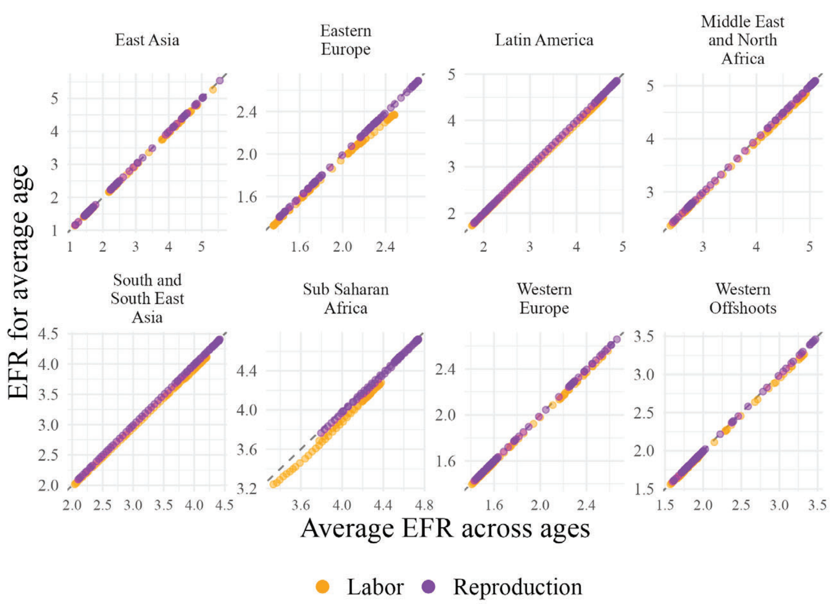
Figure A.1 demonstrates that this calculation results in a reasonable approximation of average EFR across ages in a range, i.e., Eq. (4). For different regions, the figure plot Eq. (4) versus EFR at age $\tilde{A}$ at different points in time. The purple dots show this when the age range is 15-49 and the orange dots do it for ages 15-65. Of course, the approximation at $\tilde{A}$ is not the same as the approximation at, e.g., the median of A in a range, as is used in NRR. But the additional error from using median rather than $\tilde{A}$ depends on the degree to which $p(A)$ is non-constant over age.

A.2 Midpoint age

Because calculating average age requires as much information about age-specific survival rates as calculating average EFR across ages, we show a simpler, less data-demanding approximation that uses midpoint age in lieu of average age in a range. For reproductive EFR, midpoint age is 32. For labor EFR, midpoint age is 40. Figure A.2 compares EFR at thus midpoint (y-axis) against average EFR in the relevant range.

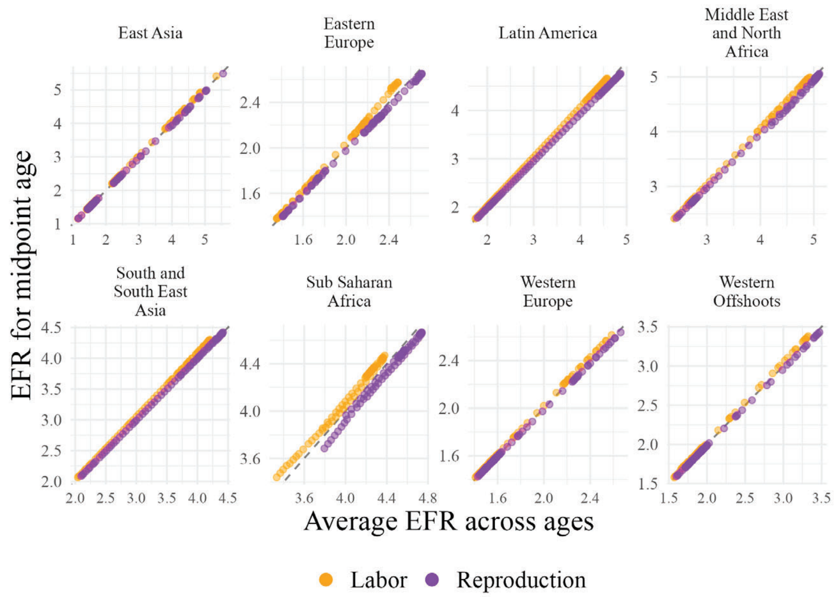
In that figure, we see that EFR at midpoint is in general a worse approximation for average EFR in Eastern Europe, Sub-Saharan Africa and Latin America. However, when we compare an approximation that uses average year versus and approximation that uses midpoint year, the difference are less stark. Table A.1 gives the average approximation error (measured in number of surviving children) for the average year and for the midpoint year approximation for each of the major regions of the world. Overall, the approximation errors are similar and small for both single-year options, typically less than 1/10 of a surviving child. Indeed, midpoint year does at least as well as average year for EFR_L. In some cases (e.g., Sub-Saharan Africa), it does better. The opposite is true for EFR_R, where the gaps are larger for midpoint year. For three of four approximate EFR statistics (i.e., other than EFR_L using midpoint age), approximations underestimate average EFR

Figure A.1: Comparing EFR at average age in a range with average EFR across a range.



Note: The x-axis shows average EFR, and the y-axis shows EFR for average age. Each dot is a population-weighted average for a given region and year. Purple dots present calculations for reproductive EFR, and yellow ones for labor EFR. We calculate a average-age EFR approximation in three steps. First, we calculate an average age for reproduction labor that is uniform across countries and years. We do this in two steps. We begin by calculating average age $\tilde{A}$ within each country and year from country life tables. Then we calculate a global average age each year as the population-weighted average across countries in each year, followed by a simple average across years of global averages by year. The result is average age of 49 in the reproductive range [15, 49] and average age 65 for the range [15, 65]. Second, we calculate the EFR for the uniform $\tilde{A}$ separately for each country and year. Third, for each year, we take a population-weighted average of EFR approximations across all countries in a region.

Figure A.2: Comparing EFR at midpoint age with average EFR across a range.



Note: The x-axis shows average EFR, and the y-axis shows EFR for midpoint age. Each dot is a population-weighted average for a given region and year. Purple dots present calculations for reproductive EFR, and yellow ones for labor EFR. Midpoint for labor age range is 40 and for the reproductive age range is 32. We calculate the EFR for the midpoint $\tilde{A}$ for each country and year in a region, and then take a population-weighted average of countries in the region for each year.

Table A.1: Average difference between EFR approximations and true EFR.

	Average Age		Midpoint	
Region	EFR_L	EFR_R	EFR_L	EFR_R
East Asia	-0.03	-0.00	0.03	-0.02
Eastern Europe	-0.05	-0.01	0.05	-0.03
Latin America	-0.05	-0.01	0.05	-0.06
Middle East and North Africa	-0.05	-0.01	0.05	-0.04
South and South East Asia	-0.06	-0.01	0.06	-0.00
Sub Saharan Africa	-0.11	-0.02	0.08	-0.07
Western Europe	-0.02	-0.00	0.02	-0.01
Western Offshoots	-0.03	-0.01	0.03	-0.02

Note: Table present the average difference between approximation using EFR for a single year and average EFR across ages. A positive difference means the approximation is larger than the average EFR. Units are the number of surviving children. Average age and midpoint age are calculated as explained in Figures A.1 and A.2.

B Current-year v. projected survival probabilities

When we formally define EFR and calculate it in the text, we use future survival probabilities of individuals born in a given year. For example, for a person born in 2010 and of age 35, we need a projection that they will live until the year 2045. We are able to use projections in our post-1950 country-year calculations because UN Life Tables contain extensive projections, even for years after the publication date of this paper.

However, there are a number of reasons that one might not use projections. Most importantly, for many birth-year age combinations future life tables do not exist. This is the case with the US race-level data we analyze in Section 3.2.2. Additionally, other fertility measures—such as NRR—use current-year survival measures, rather than projections. One might want to use techniques that allow more direct comparisons to these statistics. Finally, one might object to the idea that projections should be used at all. Perhaps parents make fertility decisions based on personal expectations that deviate from official projections or perhaps parents do not fully take into account how they expect mortality to change when making fertility decisions.

An alternative approach to using projections is to use current-year survival probabilities for different ages. For example, for a person born in 2010, one can approximate survival until age 35 by using survival to age 35 of a person born in 1975. This is the less data-demanding approach used to calculate other demographic concepts such as NRR.

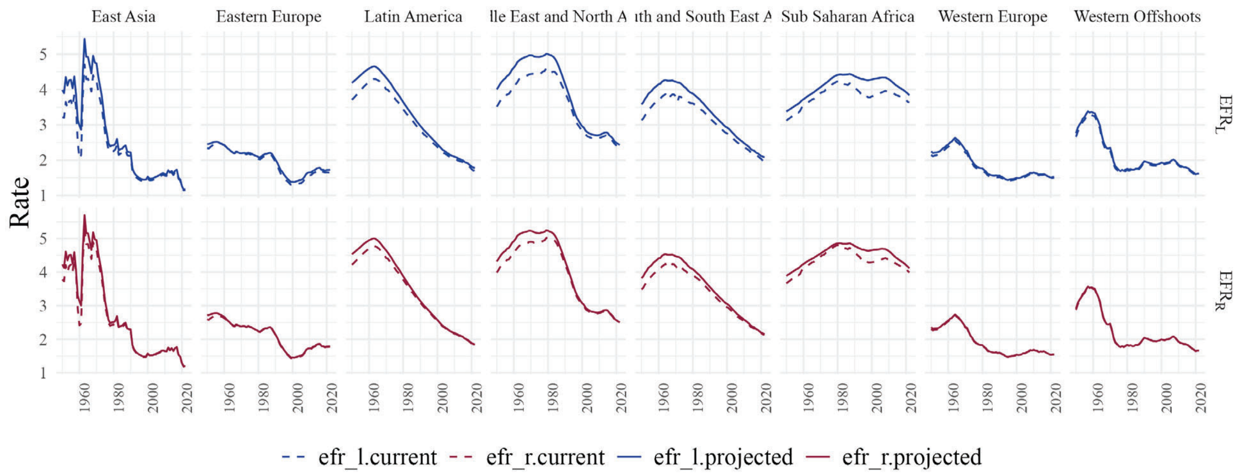
How well an approximation that uses current-year survival rather than projections to calculate EFR performs depends on differences between current year and future survival probabilities to a given age. To explore these differences, we subtract EFR calculated with projected survival rates from EFR calculated with current-year survival rates for both EFR_R and EFR_L and with (i) UN data between 1950 and 2019 and (ii) US race level data from 1800-2020. For each of these samples, we have several years where with have both projections and current-year survival rates. Figures A.3 and A.4 show the results for EFR_L and EFR_R for each of these two samples, respectively. Table A.2 quantifies the differences between EFR using projections and using current-year survival probabilities for each of the two samples.

The average differences are typically less than 1/3 of a surviving child. The average difference of EFR_L ranges from 0.05 to 0.32 surviving children depending on region or sub-population. The average for EFR_R ranges from 0.01 to 0.22 surviving children depending on region or sub-population. While the high end of these errors are more than those from single-year approximations, the errors are less than 15% of the replacement level EFR of 2. And in most cases with higher error, the EFR is usually 2 or higher.

The second takeaway from these figures and tables is that the differences between EFR when using projections and current mortality depends on the degree to which mortality is changing. When mortality is shifting a lot, such as in the early years of each sample, there are big differences between current and projected rates. When mortality is stable, the differences are negligible.

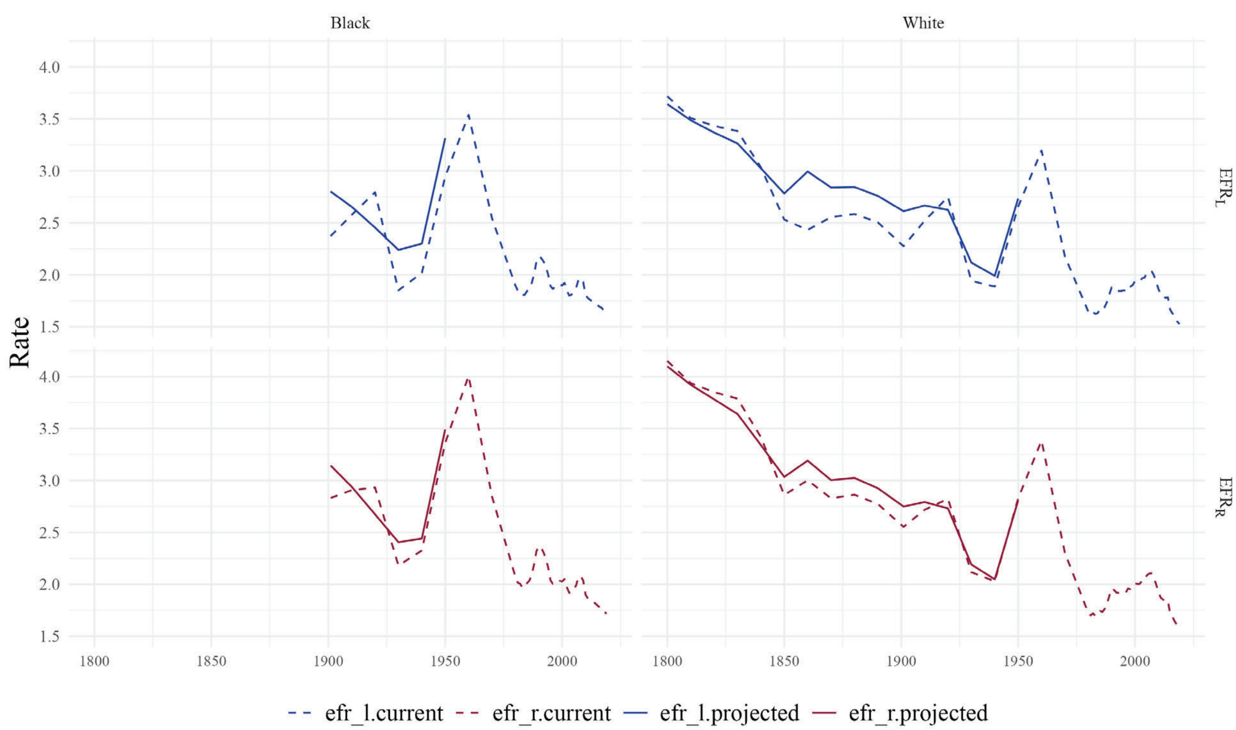
A third lesson from these tables is about the relative magnitude of errors when using the current-year survival approximation across populations. Looking across the two samples we examine, we see that the average approximation error in the US Black population is similar to the average error in South and South East Asia, and that the average approximation error in the US White population is similar to the average error in Latin America.

Figure A.3: EFR calculated using projected mortality versus current survival probabilities for 1950-2019 by region.



Note: The x-axis shows time, and the y-axis shows EFR in units of surviving children. The first row of plots shows EFR_L and the second shows EFR_R. Each column of figures shows a different sub-population indicated in column headers. Dashed lines show EFR calculated with projected survival rates, while solid lines show EFR calculated with current-year survival rates.

Figure A.4: EFR calculated using projected mortality versus current survival probabilities for US race level data.



Note: The x-axis shows time, and the y-axis shows EFR in units of surviving children. The first row of plots shows EFR_L and the second shows EFR_R. Each column of figures shows a different sub-population indicated in column headers. Dashed lines show EFR calculated with projected survival rates, while solid lines show EFR calculated with current-year survival rates.

Table A.2: Summary of the difference between EFR using current and projected mortality.

	EFR_L			EFR_R		
Group	Mean	Std. Dev.	Max.	Mean	Std. Dev.	Max.
East Asia	-0.25	0.25	-0.93	-0.13	0.16	-0.72
Eastern Europe	-0.06	0.03	-0.13	-0.02	0.03	-0.14
Latin America	-0.20	0.14	-0.48	-0.11	0.10	-0.31
Middle East and North Africa	-0.31	0.19	-0.57	-0.17	0.13	-0.36
South and South East Asia	-0.27	0.14	-0.55	-0.18	0.12	-0.37
Sub Saharan Africa	-0.28	0.10	-0.50	-0.18	0.09	-0.36
Western Europe	-0.05	0.02	-0.10	-0.01	0.01	-0.05
Western Offshoots	-0.05	0.03	-0.11	-0.01	0.01	-0.04
US Black (1901-1950)	-0.20	0.29	-0.43	-0.09	0.20	-0.31
US White (1800-1950)	-0.13	0.19	-0.56	-0.05	0.12	-0.20

Note: Table present the average difference between approximation using EFR with current year mortality and EFR with projected mortality rates. A negative difference means the current-year mortality EFR is less than the projected mortality EFR. Standard deviation and the largest difference in magnitude is also reported. The region level results are from 1950-2019.

C TFR versus TFR approximation

Data limitations require us to use approximations to calculate TFRs for historic European populations. Age-specific births are not available from Human Mortality Data. Thus, we cannot construct TFR as it is usually calculated (using 5-year age-specific fertility rates) for the analysis of historic European populations. Instead, we approximate TFR as

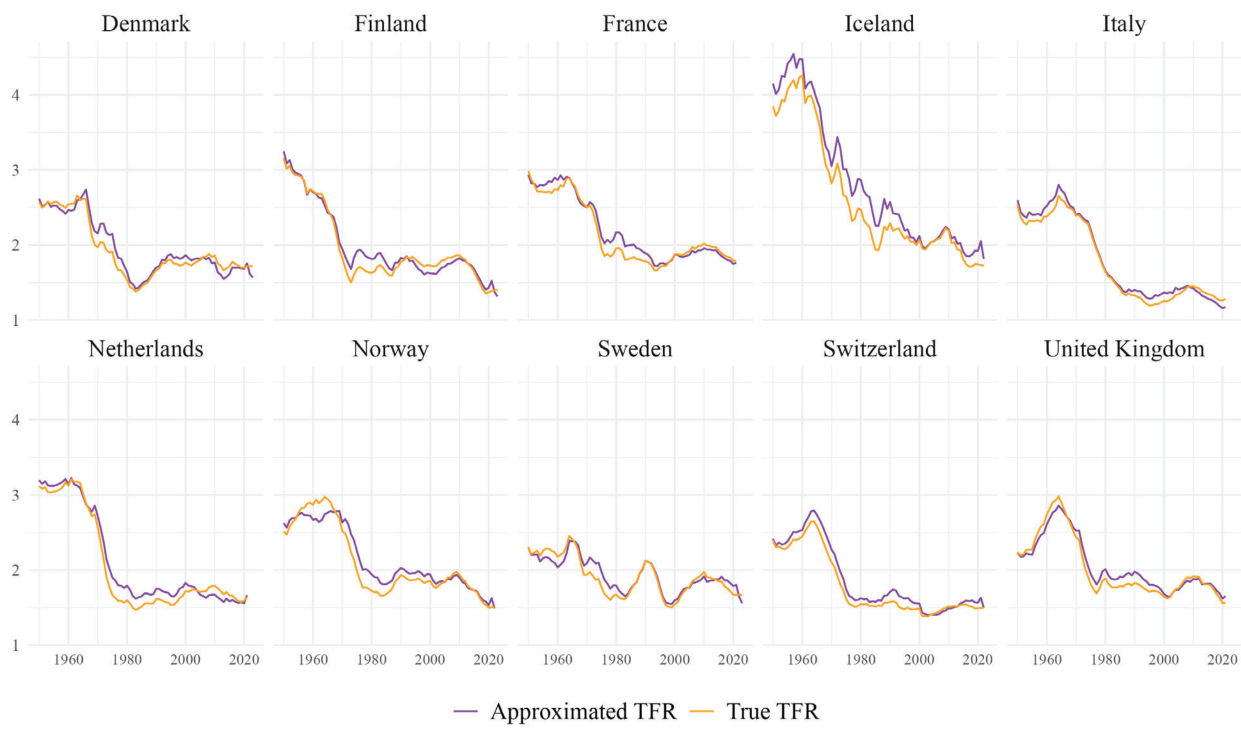
$$\text{TFR}(t) \approx 35 \cdot \frac{B(t)}{N_{fR}(t)} \tag{27}$$

where $B(t)$ is the total number of births in year t and $N_{fR}(t)$ is the number of females aged 15-49 in year t. This construction of TFR resembles our initial construction of EFR in Eq. (1).

This approximation could differ from true TFR for two reasons. First, when there are births that occur outside of the reproductive ages (15-49) then the approximation will overstate true TFR. Second, if the age-specific birth rates differ between age cohorts such that their average is not equal to $B(t)/N_{fR}(t)$, the approximation will entail error.

To asses the quality of our approximation we compare the approximation to UN-reported TFR between 1950-2019 for the countries that we use in Section 3.2. Figure A.5 shows the results of this exercise. For all countries in our historic European sample, our approximation is very close to true TFR. In particular, the average difference between approximated TFR and TFR, where positive values indicate the approximation is larger than the true value, is 0.044 births with a standard deviation of 0.084 births. We take this as evidence that our approximation works well for the whole time series.

Figure A.5: TFR and approximated TFR are close.



Note: Table plots TFR (number of children per female through her reproductive years) against time for Eurpean countries in our historical European sample. Yellow line plots true EFR based on 5-year age-specific fertility rates). Purple line plots TFR approximated with total birth in a year as in Eq. (27). We plot the United Kingdom instead of England & Wales and Scotland because the UN reports TFR for the whole country.