# Output Market Power and Spatial Misallocation*

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**Abstract**

Most product industries are local. In the U.S., firms selling goods and services to local consumers account for half of total sales and generate more than sixty percent of the nation's jobs. Competition in these industries occurs in local product markets: cities. I propose a theory of such competition in which firms have variable markups. Spatial differences in local competition arise endogenously due to the spatial sorting of heterogeneous firms. The ability to charge higher markups induces more productive firms to overvalue locating in larger cities, leading to a misallocation of firms across space. The optimal policy incentivizes productive firms to relocate to smaller cities, providing a rationale for commonly used place-based policies. I use U.S. Census establishment-level data to estimate markups and to structurally estimate the model. I document a significant heterogeneity in markups for local industries across U.S. cities. Cities in the top decile of the city-size distribution have a fifty percent lower markup than cities in the bottom decile. I use the estimated model to quantify the general equilibrium effects of place-based policies. Policies that remove markups and relocate firms to smaller cities alleviate spatial misallocation, yielding sizable aggregate welfare gains.

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# 1 Introduction

Most product industries are local. In the U.S., firms selling goods and services to local consumers account for 50% of total sales and generate 60% of the nation's total jobs.[1] Competition in these industries happens in local output markets: cities. For example, restaurants or retail stores in Manhattan do not compete with similar establishments in Chicago or Seattle. As such, competition in these industries varies at the local level. The location choice of producers is one key determinant of the strength of local competition: some cities attract productive firms that compete intensely, while other cities hardly attract any producers, leading to slack competition. When firms in these industries exert output market power, the degree of local competition becomes a crucial determinant for local welfare and resource allocation across cities.

How does imperfect competition affect the location decision of local producers? What are the welfare implications of differences in the strength of local competition across cities? Can placed-based policies remedy any resource misallocation generated by firms' output market power? In this paper, I answer these questions with four contributions. First, I develop a theory of endogenous competition across cities in which differences in market power arise due to the location choices of heterogeneous producers who compete in local markets. Second, I use the model to estimate markups by combining the consumer's demand system with the firm's production function. Third, I document significant heterogeneity in markups across U.S. cities, with cities in the top decile of city-size distribution having a markup 50% lower than cities in the bottom decile. Finally, using the estimated markups, I structurally estimate the model and quantify the welfare effects of place-based policies.

I start by presenting a theory of endogenous competition across cities, which guides the later empirical investigation. Workers choose freely where to live and work, subject to idiosyncratic location tastes. Moreover, production is divided into two sectors: local goods (non-traded) and a traded good. On the one hand, heterogeneous local goods producers choose where to operate. Once located in a city, they produce a differentiated variety using labor and commercial structures and compete monopolistically with other local producers. Furthermore, workers have Kimball preferences over the local varieties of their city. The constant-elasticity-of-substitution (CES) preferences commonly used in models of monopolistic competition (i.e., Melitz (2003)) imply a constant markup rate. In contrast, Kimball preferences allow each firm's price elasticity to vary with its position on its residual demand curve, leading to variable markups. On the other hand, perfectly competitive traded good producers are immobile across cities and produce also using labor and commercial structures. The traded good is homogeneous across locations and freely traded.

In my framework, cities differ ex-ante along two dimensions. First, they differ in a local productivity component that determines technical efficiency in producing the traded good. Second, they differ in local amenities that determine the workers' valuation for specific locations.

Local producers value two endogenous location characteristics: city size and the level of local competition. On the one hand, the size of a city determines the potential sales of local producers.

[1] See Delgado, Porter, and Stern (2015) and my calculations on Table 1.

All firms value locating in bigger cities more than locating in small ones because they can sell more. On the other hand, local competition is determined by the prices of local producers and by the cost of the inputs of production. In cities where competition is more intense, local producers attract consumers by charging lower prices. Moreover, they compete for production inputs by paying higher wages and rents. Tougher competition reduces firms' profits, and therefore, all else equal, firms prefer to locate in cities in which competition is slack.

More productive producers value relatively more locating in bigger markets. In equilibrium, larger cities are endogenously more competitive. All firms value equally locating in bigger cities where they sell more. However, more productive firms value relatively more producing in such locations. The reason is that because they can charge higher markups, they gain relatively more by the increase in sales a bigger city allows. Therefore, more productive firms self-select into larger cities where competition is endogenously tougher. In contrast, less productive have lower price-cost margins. In turn, it is optimal for less productive firms to locate in smaller cities where they sell less, but the market is less competitive.

In equilibrium, two opposite forces determine markup differences across cities. On the one hand, a "competition force" pushes markups in bigger cities down. As bigger markets are endogenously tougher, firms operating in those markets must charge lower markups. On the other hand, there is a "selection force" that pushes markups up in bigger markets: bigger cities attract more productive firms with higher markups. The relative strength of these two forces determines whether bigger cities have lower or higher markups in equilibrium. When the dispersion in firm productivity is low compared to the dispersion in city fundamentals, the competition force dominates, and bigger cities have lower markups. In contrast, when firms' productivity dispersion is high compared to the dispersion in city fundamentals, selection of bigger cities is prevalent enough that the resulting markups are higher than those in smaller cities.

The spatial equilibrium allocation is inefficient because of two externalities firms create when entering a city. On the one hand, introducing a new variety raises consumer surplus as workers value variety. However, firms can only partially appropriate the gain in consumer surplus into their profits, which leads to insufficient entry. This is a positive externality that I refer to as the "variety gains" externality.[2] On the other hand, firms impose a negative externality on the incumbent firms as by reducing the consumption of existing varieties' consumption, which reduces incumbents' profits. This externality leads to excessive entry because firms do not internalize their negative effect on other firms' profits. This is a standard "business stealing" externality. Whether in a city there is excessive, insufficient, or efficient entry is determined by which of these externalities dominate.[3]

The variety-gains and the business-stealing externalities make productive firms over-concentrate in bigger cities. In the spatial equilibrium, the gains from additional variety are larger in cities in

[2] In other words, firms are not correctly compensated for the consumer gains they generate by entering a market.

[3] These externalities are not specific to the framework considered here. As recognized early by Mankiw and Whinston (1986), these externalities are present in models of free entry and differentiated varieties. However, in the standard model of monopolistic competition with CES preferences, these externalities are constant and exactly offset each other, leading to efficient entry.

which competition is slack: smaller cities. Smaller locations can only attract a few local producers, and the ones that decide to operate there are of low productivity. In contrast, the business stealing effect is higher in high-profit locations that attract more productive firms: bigger cities. Therefore, in equilibrium, there is excessive entry in bigger cities and insufficient entry in smaller ones. There is spatial misallocation as more productive firms over-concentrate in bigger markets, and aggregate welfare can increase by reallocating them to smaller markets. An utilitarian social planner chooses an optimal policy that incentivizes productive firms to relocate to smaller locations, providing a rationale for commonly used place-based policies.

In the second part of the paper, I use the model estimate markups. The empirical approach combines the demand structure from the model with the firm's production function.[4] Specifically, the firm's first-order condition with respect to labor uncovers a relationship between the markup, the labor output elasticity, and the labor cost-share of sales. I use the demand system to construct a control function for the markup, which depends on the firm's local sales share and price indices. Using a semi-parametric polynomial approximation for the markup function, the estimating equation results in a heterogeneous-slope model in which markups are identified using within-city variation in the sales market share and the labor cost share of revenue.

In the third part of the paper, I implement the proposed empirical strategy and estimate markups for establishments in local industries in the U.S.[5] I use data from the Longitudinal Business Database (LBD) to construct establishments' labor cost-shares of sales and data from the Economic Censuses (EC) to construct establishments' local sales share. The estimated markup level is similar to those obtained using other markup estimation methods, although they exhibit slightly higher dispersion. Then, I aggregate establishment-level markups into the city-level markup using the aggregation implied by my model.[6]

I find large heterogeneity in markups for local industries across U.S. cities. Cities in the top decile of the city-size distribution have a 50% lower markup than cities in the bottom decile. This fact is not specific to a particular year, as there is high persistence in the heterogeneity of markups across cities throughout the years. Also, the negative relationship between city size and city markup is also robust to different city definitions.[7]

In the last section of the paper, I estimate the model for the 3080 counties of the continental U.S. The estimation of the main parameters of the model is done in two blocks. The demand parameters and the firm's output elasticities are estimated in the first block using a Generalized Method of Moments (GMM). The model produces estimating equations for the firms' technology parameters and a recursive scheme for the demand-side parameters. In the second block, I estimate the firms' productivity distribution and the economy-wide entry cost using the Simulated Method of Moments (SMM). I target the establishment-size distribution across counties and the economy-wide aggregate markup. Regional fundamentals are recovered by exactly matching population and

[4] I consider the class of Homotheticity with Direct Implicit Additivity (HDIA) preferences defined and conceptualized by Matsuyama and Ushchev (2017).

[5] To classify establishments as Local establishments, I use the classification of Delgado, Porter, and Stern (2015).

[6] The city-level aggregate markup is a sales-weighted harmonic mean of the establishment-level markups. This is the same definition as in Yeh, Macaluso, and Hershbein (2022) and Edmond, Midrigan, and Xu (2023).

[7] Formally, the fact is robust to define a city as a county or as a commuting zone.

average wages for each county. Over-identification exercises support estimates of crucial parameters and highlight the model's ability to match important non-targeted moments.

I use the estimated model to conduct one counterfactual exercise that evaluates the effect of the optimal policy. The policy that achieves the first-best allocation takes the form of a non-linear location-specific subsidy per total production. The subsidy effectively eliminates markups by incentivizing firms to produce at marginal cost. Moreover, this policy correctly compensates firms as net profits after transfers coincide with each firm's consumer surplus in a given city. Then, marginally productive producers in bigger cities find it optimal to reallocate to smaller cities where they earn higher profits by generating a higher consumer surplus. This policy achieves the first-best equilibrium allocation by removing markups and effectively reallocating local producers from big to smaller cities.

The optimal policy has sizable quantitative implications for the distribution of economic activity. First, the policy is effective in reallocating productive establishments to small counties. Local total factor productivity (TFP) in small-sized counties increases by 10%. Productivity improvements in small counties only modestly affect productivity in larger counties. Places like Cook County (Chicago) only experience a 5% reduction. Second, prices of local varieties drop in all counties. However, because smaller counties originally had larger markups and experience an influx of more-productive, low-price firms, they experience more considerable price reductions: in Manhattan, prices reduce by 40%, whereas Wilcox County, GA, experiences a reduction three times larger. In the aggregate, the policy yields a welfare gain of 2.36%.

**Related literature.** This paper contributes to five strands of the literature. The first strand is the recent body of work examining the aggregate implications of markups. Edmond, Midrigan, and Xu (2023) quantify markups' aggregate welfare cost using a model encompassing monopolistic competition with Kimball (1995) preferences and oligopolistic competition with nested-CES. I use their monopolistic competition market structure and extend it into a spatial framework with many output markets. I depart from their work by quantifying the markup welfare costs through a new channel: the inefficient location of firms. Other studies have analyzed the markup implications for business dynamism, including the labor share, capital share, R&D expenditures, and product creation (De Loecker, Eeckhout, and Unger (2020), Aghion et al. (2023), Akcigit and Ates (2023)). I depart from these studies by studying the effect of markups on regional aggregates, such as local productivity and prices.

The second strand of the literature this paper relates to is the one on misallocation. Similar to Edmond, Midrigan, and Xu (2023), in my model, more productive firms charge higher markups. This creates misallocation in the form of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). Nevertheless, rather than focusing on misallocation across firms, I focus on misallocation across places. In particular, I show how imperfect competition leads firms to locate inefficiently across cities. In turn, aggregate welfare increase by relocating firms across places.

Third, this paper is related to the literature on firm sorting. Gaubert (2018) and Bilal (2023) consider models of firm sorting through agglomeration forces and labor market frictions. I depart from these studies by considering a framework in which firms sort through competitive price

pressures. The policy implications of my framework align with those in Bilal (2023), who also finds policies relocating firms to smaller locations beneficial. Matsuyama and Ushchev (2022) also consider a setting in which firms sort through competitive price pressures. However, in their framework, population across markets is exogenously given. I build on this framework and consider a setting where the population is endogenously determined, allowing me to study additional general equilibrium forces interacting with the firm location.

The fourth strand of the literature I contribute to is the one studying markups across space. Hottman (2021) and Anderson, Rebelo, and Wong (2018) study markups across cities for the Retail sector. On the one hand, Hottman (2021) builds an oligopolistic competition model and estimates it using scanner data. He finds that markups are lower in more populous cities. On the other hand, Anderson, Rebelo, and Wong (2018) use gross margins as a proxy for markups and show that wealthier cities have higher markup levels. I depart from these studies in two ways. First, the scope of my markup estimation extends beyond the Retail sector to all local industries in the U.S. Similar to Hottman (2021), I find that markups in Retail are lower in bigger cities. Second, I use a different methodology to estimate markups. Compared to Anderson, Rebelo, and Wong (2018), my alternative methodology allows me to construct a direct measure of markups rather than relying of a proxy. Furthermore, compared to Hottman (2021), my procedure remains agnostic of the market structure under which firms compete, requiring only an assumption on the consumers' preferences.

Finally, this paper relates to the literature on local competition and cannibalization in output markets. On the one hand, Hsieh and Rossi-Hansberg (2023), Kleinman (2023)), and Oberfield et al. (2023) study the location of multi-establishment firms across local output markets. I differ from these studies by considering a framework in which firms have endogenous and variable output market power. On the other hand, Rossi-Hansberg, Sarte, and Trachter (2020) documents secular trends in local output market concentration. My paper complements this study by empirically providing direct measures of output market power across local markets.

The rest of the paper is organized as follows. Section 2 lays out the theoretical framework. Section 3 explores the efficiency properties of the model, emphasizing the spatial misallocation of firms. 4 presents the empirical analysis investigating markups across cities and model predictions. Section 5 details the quantitative analysis in which I estimate the model and quantify the welfare gains of place-based policies. Finally, Section 6 concludes.

# 2 Model

This section develops a theory of spatial differentials in local competition. The theory abstracts from dynamics, describing a long-run steady state of the economy. Production takes place in locations I call cities.

## 2.1 Environment

**Geography.** There is a continuum of cities indexed by $c \in [0,1]$. These cities differ in their local productivity, $a(c) \in [\underline{a}, \overline{a}]$, and their local amenities $b(c) \in [\underline{b}, \overline{b}]$. These local characteristics are distributed with a cumulative distribution function $F(c) \equiv F(a(c), b(c))$, with density $f(c) \equiv f(a(c), b(c))$. Hence, location $c$ is characterized by the pair $(a(c), b(c))$ rather than its particular name. Furthermore, every location has a fixed land supply, which is used for building housing and structures.[8]

**Workers Preferences.** The whole economy is populated by $\overline{L}$ freely mobile identical workers, indexed by $i$. Each worker has one unit of labor, which is supplied inelastically. Worker $i$ observes a collection of idiosyncratic location-specific preference shocks, $\varsigma_i(c)$, and decides her location of work and residence. When locating in c, worker $i$ derives utility from consuming a bundle of local varieties $Y(c)$, housing, $H(c)$, and a freely traded good, $Q(c)$, according to:

$$U_i(c) = b(c) \left(\frac{Y(c)}{\eta}\right)^{\eta} \left(\frac{H(c)}{\alpha}\right)^{\alpha} \left(\frac{Q(c)}{1-\eta-\alpha}\right)^{1-\eta-\alpha} \varsigma_i(c), \tag{1}$$

where $\eta$ and $\alpha$ are the expenditure shares on local goods and housing. Consumers have symmetric Kimball preferences (Kimball (1995)) over local varieties. These preferences are in the Homothetic with Direct Implicit Additivity (HDIA) family of preferences defined by Matsuyama and Ushchev (2017). Under these preferences, the per-capita consumption of the bundle of local goods, $Y(c)$, is implicitly given by

$$\int_z \Upsilon\left(\frac{y(z,c)}{Y(c)}\right) dG_c(z) = 1, \tag{2}$$

where $y(z,c)$ is the per-capita consumption of a local variety produced by a firm with productivity $z$, $G_c(\cdot)$ is local producers productivity distribution in $c$, and $\Upsilon(\cdot)$ is a strictly increasing and concave function satisfying $\Upsilon(0) = 0$.[9] CES preferences are special case of (2) when $\Upsilon(x) = x^{\frac{\sigma-1}{\sigma}}$.

Kimball preferences have three advantages. First, they can generate cross-sectional variation in markups, the central object of this paper. Second, they are homothetic. Therefore, they allow us to focus on markup differences across cities due to price competition pressures and not from potential income effects.[10] Third, despite their flexibility, they remain tractable enough to characterize the model's equilibrium uniquely. In models with similar preferences like nested CES (as in Atkeson and Burstein (2008)), problems of multiple equilibria often arise. This becomes more challenging when producers have an entry decision per market, as is the case in this study. Appendix C.2

[8] I normalize the supply to be one in every location.

[9] This notation previews that in equilibrium, two firms of the same productivity and located in the same city make the same pricing and production decisions. Therefore, consumers consume the same amount of each variety produced by each of these firms.

[10] Although this is an important channel to study, it is left for future research. See Melitz and Ottaviano (2008) and Combes et al. (2012) for settings in which firms have endogenous variable markups due to non-homothetic linear preferences.

shows that the theoretical results hold under Homothetic with a Single Aggregator (HSA) demand systems.[11]

The idiosyncratic preferences draw $\varsigma_i(c)$ is assumed to be independent, identically distributed across individuals and cities, and following a Frechet distribution with shape parameter $\theta$.[12]

**Local Varieties.** A mass $M_e$ of potential entrants pays an entry cost $c_e$ to learn their productivity $z$. This productivity has common distribution with cumulative distribution function $G_z(\cdot)$, density function $g_z(\cdot)$, and connected support $[\underline{z}, \overline{z}]$. After learning their productivity, firms choose a city to produce and sell. This location decision determines the set of locally available varieties. Within a city, local producers compete in a monopolistically competitive fashion and produce according to a Cobb-Douglas production function

$$y(z,c) = z\, l(z,c)^{\beta} s(z,c)^{1-\beta}, \tag{3}$$

where $l(z,c)$ is labor, and $s(z,c)$ is commercial structures (buildings). Firms pay a common wage of $W(c)$ and commercial structures rent $R(c)$ in city $c$.

**Traded Good.** A perfectly competitive representative firm produces the homogeneous traded good in every location. This good is freely traded and used as the *numeraire*. Similarly to the local varieties technology, the traded good is produced by combining labor and commercial structure according to a Cobb-Douglas production function given by

$$Q^T(c) = a(c)\ \left(L^T(c)\right)^{\gamma} \left(S^T(c)\right)^{1-\gamma}, \tag{4}$$

where $Q^T(c)$ denotes the total production of traded good in city $c$, $L^T(c)$ is the traded good total employment, and $S^T(c)$ is the traded good total commercial structures demand. We use different notation for the traded good quantities produced in $c$, $Q^T(c)$, and the traded good workers demand, $Q(c)$. Because of trade across cities, these two quantities are different. Traded good producers compete in the same local inputs market with the local varieties producers, paying a wage $W(c)$ and a price for commercial structures $R(c)$.

**Land Developers.** In every city, competitive land developers use the traded good to produce housing and commercial structures according to the isoelastic production function:

$$\overline{H}(c) = \left(\frac{1+\phi}{\phi}\overline{Q}(c)\right)^{\frac{\phi}{1+\phi}},$$

where $\overline{H}(c)$ is the total supply of buildings in city $c$ (housing and commercial structures), and $\overline{Q}(c)$ is the amount of traded good used for buildings. I assume that land developers use their profits to consume the final good only. However, for the counterfactual exercises, I consider an alternative

[11] This class of demand systems was defined and characterized by Matsuyama and Ushchev (2017) .

[12] See Bilal (2023) Appendix G.4. for a discussion of how to extend discrete choice results to a framework with a continuum of locations.

formulation in which land developers' profits are aggregated into a national portfolio and rebated back to workers as a flat labor subsidy.[13]

## 2.2 Worker's Consumption and Location Decisions

We start by characterizing the workers' optimal consumption and location decisions. Workers solve this problem in two steps: first, conditional on locating in c, they solve for the optimal consumption quantities, which determines local utility. Then, they choose where to locate, conditional on local utility and the realization of their preference shocks.

When choosing how much of the local varieties, housing, and traded good to consume, workers face the budget constraint

$$\mathbb{P}(c)Y(c) + R(c)H(c) + Q(c) = W(c) \tag{5}$$

where $\mathbb{P}(c)$ is the price of the bundle of local varieties, $R(c)$ is the housing price, and where we used the fact that the traded good is used as the numeraire.[14] The homotheticity of the Kimball preferences guarantees the existence of a price index for the bundle of local varieties. Therefore, workers maximize (1) subject to (2) and (5). The per capita consumption of local varieties, housing, and the traded good that result from this maximization are given by

$$Y(c) = \frac{\eta W(c)}{\mathbb{P}(c)}, \qquad H(c) = \frac{\alpha W(c)}{R(c)}, \qquad \text{and} \qquad Q(c) = (1-\eta-\alpha)W(c). \tag{6}$$

Appendix A.1 shows that the per-capita consumption of an individual variety $y(z,c)$ is, in turn

$$\frac{y(z,c)}{Y(c)} = \varphi\left(\frac{p(z,c)}{\mathbb{D}(c)}\right). \tag{7}$$

where $p(z,c)$ is the price of a variety produced by a firm with productivity $z$ in city $c$, $\varphi(\cdot) \equiv (\Upsilon')^{-1}(\cdot)$, and $\mathbb{D}(c)$ is a price index implicitly defined by

$$\int_z \Upsilon\left(\varphi\left(\frac{p(z,c)}{\mathbb{D}(c)}\right)\right) dG_c(z) = 1. \tag{8}$$

The expression in (7) is the residual demand curve faced by local variety producers. For an individual firm, changes in other firms' prices are summarized by the price index $\mathbb{D}(c)$. In other words, firms in every location compete against the price index $\mathbb{D}(c)$ when choosing their optimal price. Therefore, $\mathbb{D}(c)$ captures the degree of local competition, and I call it the *competition price*

[13] This can be interpreted as workers having a share in the national portfolio which is increasing in the level of income. See Redding and Rossi-Hansberg (2017) Section 2.7.3 for a general discussion of rebate schemes in quantitative spatial models.

[14] Throughout the text, I use the blackboard bold font notation for indices. There are three of them: the ideal price index $\mathbb{P}(c)$, the price competition index, $\mathbb{D}(c)$, and the competition index, $\mathbb{C}(c)$.

*index*. In contrast, the *ideal price index*, $\mathbb{P}(c)$, which is the price of the bundle of local varieties, is given by

$$\mathbb{P}(c) = \int_z p(z,c)\varphi\left(\frac{p(z,c)}{\mathbb{D}(c)}\right) dG_c(z). \tag{9}$$

Two price indices then characterize the Kimball demand system. The competition price index $\mathbb{D}(c)$ mediates the relative consumption of different varieties, whereas the ideal price index $\mathbb{P}(c)$ determines the consumption of the overall bundle $Y(c)$ relative to other goods. In the particular case of CES, these price indices are proportional.

Workers consumption decisions (6) imply that the indirect utility of worker $i$ in city $c$ is given by

$$U_i(c) = u(c)\varsigma_i(c), \qquad \text{with} \qquad u(c) \equiv b(c)\frac{W(c)}{\mathbb{P}(c)^{\eta}R(c)^{\alpha}}, \tag{10}$$

where $u(c)$ is the mean utility of workers residing in $c$. After observing the elements of $u(c)$ and the collection of idiosyncratic location preference shocks, $\varsigma_i(c)$, workers choose the location $c$ that maximizes $U_i(c)$. The Frechet assumption implies that the share of workers residing in $i$ is equal to

$$\frac{L(c)}{\overline{L}} = \left(\frac{u(c)}{\overline{U}}\right)^{\theta}, \qquad \text{with} \qquad \overline{U} = \left[\int_c u(c)dF(c)\right]^{\frac{1}{\theta}}. \tag{11}$$

The expression (11) is the supply of workers in city $c$. When $\theta$ is larger, the idiosyncratic preference shocks, $\varsigma_i(c)$, are less dispersed, and therefore cities become closer substitutes. In equilibrium, (11) implies that a higher $\theta$ makes workers in $c$ more sensitive to changes in the local utility level, $u(c)$. Note that, all else equal, cities with higher wages and amenities are more desirable for workers. Similarly, cities with lower housing rents and lower local varieties prices attract a higher share of workers.

## 2.3 Local Varieties Production and Location Decisions

Firms producing local varieties pay the entry cost $c_e$ to learn their productivity. Then, they choose a city $c$ to set up production, and choose the price that maximizes profits. We solve this problem backwards.

### 2.3.1 Profit Maximization: Optimal Markup

Local varieties producers in $c$ set their optimal price given their own productivity, $z$, and location aggregates, $Y(c)$, $\mathbb{D}(c)$, $W(c)$, and $R(c)$.

The production function in (3) implies that the marginal cost for firm $z$ of producing one unit of

output in location c is equal to $\nu(W(c)^{\beta}R(c)^{1-\beta})/z$, where $\nu$ is a constant depending solely on $\beta$.[15] Therefore, firm $z$ located in $c$ chooses $p(z,c)$ to maximize:

$$\Pi(z,c) = \max_{p(z,c)} \left[ p(z,c)y(z,c) - \frac{\nu W(c)^{\beta}R(c)^{1-\beta}}{z} y(z,c) \right] L(c) \qquad \text{s.t} \qquad (7). \tag{12}$$

The profit function in (12) scales with the number of workers in $c$, because larger cities represent a larger customer base. The first-order condition of this problem is given by

$$\frac{p(z,c)}{\mathbb{D}(c)} \left[ 1 - \frac{1}{\sigma\left(\frac{p(z,c)}{\mathbb{D}(c)}\right)} \right] = \frac{\mathbb{C}(c)}{z}, \tag{13}$$

where $\sigma(\cdot)$ is the price-elasticity of demand implied by the residual demand curve (7),

$$\sigma\left(\frac{p(z,c)}{\mathbb{D}(c)}\right) \equiv -\frac{\partial \log y(z,c)}{\partial \log p(z,c)} = \frac{-\frac{p(z,c)}{\mathbb{D}(c)}\varphi'\left(\frac{p(z,c)}{\mathbb{D}(c)}\right)}{\varphi\left(\frac{p(z,c)}{\mathbb{D}(c)}\right)}, \tag{14}$$

and $\mathbb{C}(c)$ is a competition index summarizing the local competitive pressures:

$$\mathbb{C}(c) \equiv \frac{\nu W(c)^{\beta}R(c)^{1-\beta}}{\mathbb{D}(c)} \tag{15}$$

Importantly, this index accounts for competition in the local input market and the price competition from other local producers through the competition price index, $\mathbb{D}(c)$. When there is more intense competition in the input market, wages or commercial structures rents are high; therefore, $\mathbb{C}(c)$ increases. Similarly, when other local producers in location $c$ set lower prices, profits for potential entrants decrease. This is captured by a lower price index $\mathbb{D}(c)$, which ultimately translates into a higher $\mathbb{C}(c)$.

In the main text, I assume a particular parametrization of the Kimball aggregator $\Upsilon(\cdot)$ to facilitate the exposition. For parsimony, as it is standard in the literature, I assume the aggregator takes the Klenow and Willis (2016) functional form. Nonetheless, in Appendix C.1 I provide conditions for more general aggregators for which the main theoretical predictions of the model hold.

**Assumption 1** (Parametric form of Kimball aggregator)**.** *The Kimball aggregator $\Upsilon(\cdot)$ is given by the Klenow and Willis (2016) functional form. Under this specification, the residual demand curve (7) takes the form:*

$$\log \frac{y(z,c)}{Y(c)} = \frac{\overline{\sigma}}{\varepsilon} \log \left( 1 + \varepsilon \log \frac{\overline{\sigma}-1}{\overline{\sigma}} - \varepsilon \log \frac{p(z,c)}{\mathbb{D}(c)} \right),$$

[15] Formally, $\nu \equiv \frac{1}{\beta^{\beta}(1-\beta)^{1-\beta}}$.

*with* $\overline{\sigma} > 1$, *and* $\varepsilon > 0$ *parameters.*

Under Assumption 1, the price elasticity of demand can be written as:

$$\sigma\left(\frac{p(z,c)}{\mathbb{D}(c)}\right) = \frac{\overline{\sigma}}{1 + \varepsilon \log \frac{\overline{\sigma}-1}{\overline{\sigma}} - \varepsilon \log \frac{p(z,c)}{\mathbb{D}(c)}}. \tag{16}$$

Equation (16) illustrates two advantages of the Klenow and Willis (2016) functional form specification. First, when $\varepsilon = 0$, one obtain the special CES case as $\sigma\left(p(z,c)/\mathbb{D}(c)\right) = \overline{\sigma}$. Second, whenever $\varepsilon > 0$, the price elasticity of demand is an increasing function of the relative price. This is often referred to as Marshall's second law of demand (Marshall (1890)).[16] Intuitively, workers become less price sensitive when facing lower prices. Therefore, producers that charge a lower price relative to the price index $\mathbb{D}(c)$ face a less elastic demand.

Conditional on locating in $c$, firm $z$ chooses a profit-maximizing relative price that is given by the first-order condition (13):

$$\begin{aligned} \frac{p(z,c)}{\mathbb{D}(c)} &= \psi\left(\frac{\mathbb{C}(c)}{z}\right), \\ &= \frac{\overline{\sigma}}{\varepsilon} \frac{\frac{\mathbb{C}(c)}{z}}{\Omega\left(\lambda \frac{\mathbb{C}(c)}{z}\right)}, \end{aligned} \tag{17}$$

where $\lambda$ is a constant, $\Omega(\cdot)$ is the main branch of the Lambert-W function, and $\psi(\cdot)$ is strictly increasing.[17] Similar to the frameworks in Atkeson and Burstein (2008) and Amiti, Itskhoki, and Konings (2019), firms "price-to-market" by choosing an optimal price relative to their competitor's prices summarized by the price index $\mathbb{D}(c)$. Similarly, firms choose an optimal relative quantity, which is obtained by combining (7) and (17):

$$\begin{aligned} \frac{y(z,c)}{Y(c)} &= \varphi\left(\psi\left(\frac{\mathbb{C}(c)}{z}\right)\right), \\ &= \left[\overline{\sigma} - \varepsilon \Omega\left(\lambda \frac{\mathbb{C}(c)}{z}\right)\right]^{\frac{\overline{\sigma}}{\varepsilon}} \end{aligned}$$

On the other hand, the profit-maximizing markup is given by (16) and (17) through the standard Lerner formula:

[16] See Melitz (2018) for a discussion of Marshall's different notions of demand.

[17] $\lambda \equiv \frac{\overline{\sigma}}{(\overline{\sigma}-1)\varepsilon} \exp\left(\frac{\overline{\sigma}-1}{\varepsilon}\right)$, and $\Omega(x)$ is implicitely defined by $x = \Omega(x)\exp(\Omega(x))$.

$$\mu\left(\frac{\mathbb{C}(c)}{z}\right) = \frac{\overline{\sigma}}{\varepsilon\Omega\left(\lambda\frac{\mathbb{C}(c)}{z}\right)} \geq 1. \tag{18}$$

Equation (18) reveals the two forces that determine the markup for a local producer $z$ in city $c$. First, more intense local competition captured by $\mathbb{C}(c)$ implies a lower markup.[18] Thus, across cities, firm $z$ will charge a lower markup in cities in which competition is more intense. Intuitively, price pressures from other producers and competition in the local input markets restrain firms from charging higher markups. Second, within a city, producers with higher productivity will charge higher markups. Because more productive producers charge lower relative prices (see (17)), they face a less elastic demand and, therefore, charge higher markups. I call this positive relationship between firms' markup and productivity, "pricing complementarity". Unlike monopolistic competition models with CES preferences in which markups are constant, in the framework considered here, all else constant, more productive firms charge higher markups.

Finally, Appendix A.2 shows that optimal labor and commercial structures demands are given by:

$$\begin{aligned} l(z,c) &= \beta\frac{\psi\left(\frac{\mathbb{C}(c)}{z}\right)\varphi\left(\psi\left(\frac{\mathbb{C}(c)}{z}\right)\right)\mathbb{D}(c)Y(c)}{\mu\left(\frac{\mathbb{C}(c)}{z}\right)W(c)}, \\ s(z,c) &= (1-\beta)\frac{\psi\left(\frac{\mathbb{C}(c)}{z}\right)\varphi\left(\psi\left(\frac{\mathbb{C}(c)}{z}\right)\right)\mathbb{D}(c)Y(c)}{\mu\left(\frac{\mathbb{C}(c)}{z}\right)R(c)} \end{aligned} \tag{19}$$

Total labor employed by local producers, $L^N(c)$, and total structures demanded by local producers, $S^N(c)$, are given by:

$$L^N(c) = \int_z l(z,c)dG_c(z), \qquad \text{and} \qquad S^N(c) = \int_z s(z,c)dG_c(z) \tag{20}$$

### 2.3.2 Location Decision

Now we turn to analyze the location decision of the local varieties producers. A producer with productivity $z$ contemplates the potential profits in every city and chooses the city that delivers the highest profits.

Let $M(c)$ denote the total expenditure of local varieties in location $c$: $M(c) \equiv \mathbb{P}(c)Y(c)L(c)$. We refer to $M(c)$ as the *size* of the city $c$ as it measures local producers' potential revenue in a particular location.

We can use (17) to write into the firms profits gives firms' $z$ overall potential profits:

[18] The Lambert-W function $\Omega$ is strictly increasing.

$$\Pi(z) = \max_{c} \underbrace{\log M(c)}_{\text{Market size}} + \underbrace{\log \frac{\psi\left(\frac{\mathbb{C}(c)}{z}\right)\varphi\left(\psi\left(\frac{\mathbb{C}(c)}{z}\right)\right)}{\mathbb{P}(c)/\mathbb{D}(c)}}_{\text{Market Share}} - \underbrace{\log \frac{\mu\left(\frac{\mathbb{C}(c)}{z}\right)}{\mu\left(\frac{\mathbb{C}(c)}{z}\right) - 1}}_{\substack{\text{Fraction of sales} \\ \text{going to inputs}}} \tag{21}$$

Equation (21) reveals three forces that shape the location decision of local varieties producers. The first, market size, reflects the total expenditure of local varieties in the city $c$. All producers prefer bigger cities because potential revenue is higher.

The second term, market share, indicates how much of the total revenue in a given city each firm can appropriate. The market share of firm $z$ depends on the firm's relative price, $\psi\left(\mathbb{C}(c)/z\right)$, and on the demand's price indices, $\mathbb{D}(c)$ and $\mathbb{P}(c)$.[19] Producers charging a lower relative price have a higher market share, which allows them to capture a larger fraction of the total expenditure on local varieties and, therefore, have higher sales. The ratio $\mathbb{P}(c)/\mathbb{D}(c)$ captures the sales of the other local producers in $c$.[20] Producers prefer locations in which they can have a higher market share.

Finally, the last term (21) encodes how much of the firms' sales are going to their profits and how much goes to pay the inputs of production. In the particular CES case, this last term is constant, which implies that firms' profits are always proportional to firms' sales. Nevertheless, profits are no longer proportional to sales once one departs from CES. Because each firm chooses its optimal markup, how much revenue goes to pay the production inputs varies across producers. In particular, high markup producers turn a larger fraction of the sales into revenue. All else equal, firms value locations in which they can charge higher markups. Nevertheless, note that in locations where the competition is tougher, higher $C(z)$, all firms charge lower markups and, therefore, appropriate a smaller fraction of sales into profits. As it will be clear later, this characteristic of the profit function will induce firms of different productivities to sort into different markets.

## 2.4 Traded Good Producers

Now, we turn to characterize the traded good producers problem. Recall that these producers are immobile and only make production decisions.[21] The production function (4) implies that the marginal cost of the perfectly competitive producers is $\varrho(W(c)^{\gamma}R(c)^{1-\gamma})/a(c)$, where $\varrho$ is a constant depending on $\gamma$.[22] Therefore, the zero-profit condition in every city implies:

$$a(c) = \varrho W(c)^{\gamma}R(c)^{1-\gamma}. \tag{22}$$

[19] As first recognized by Matsuyama and Ushchev (2017), the distinct characteristic of the HDIA preferences is that the firm's market share is given by the two price indices of the demand system. As shown in Appendix C.2, under HSA preferences, the market share can be characterized by only one price index.

[20] Indeed, note that (9) and (17) imply that: $\mathbb{P}(c)/\mathbb{D}(c) = \int_z \psi\left(\frac{\mathbb{C}(c)}{z}\right)\varphi\left(\psi\left(\frac{\mathbb{C}(c)}{z}\right)\right) dG_c(z)$.

[21] The retrained mobility of the traded good producers is immaterial for the model predictions. As these producers make zero profits in every location, they would be indifferent between locating among any cities.

[22] $\varrho \equiv \frac{1}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$.

In equilibrium, the cost of production in more productive cities will be higher. This translates into higher wages and housing/commercial structures rents in such locations. Note that (22) has implications for the local good producers' production and location decisions. Because traded good and local producers compete in the same labor and housing markets, local producers in more productive cities face tougher competition in the input market, embedded in higher input costs. As seen from (15), tougher competition in the input market leads to a higher overall competition index.

Finally, the production function (4) implies that total labor labor demand and total commercial structures demand from traed good producers in city $c$ are given by:

$$L^T(c) = \frac{\gamma Q^T(c)}{W(c)}, \qquad S^T(c) = \frac{(1-\gamma)Q^T(c)}{R(c)}. \tag{23}$$

Because traded good producers earn zero profits in equilibrium, each production input receives a fraction of the total sales given by their output elasticities.

## 2.5 Sorting Characterization

In equilibrium, cities are characterized by a combined index. Recall that, initially, a city is described by a pair of productivity of the traded good and amenities, $a, b$. Appendix B.1 shows that:

$$c(a,b) = a^{\frac{1+\theta(1-\eta\beta)}{\gamma}} b^{\theta} \tag{24}$$

is a local sufficient statistic for the model's outcomes. In particular, the city-level objects that determine the firms' location decisions in (21) depend solely on $c = c(a,b)$. Intuitively, local amenities govern local population (see (11)), and local productivity determines local wages (see (22)). Thus, city size depends on a combined index of these two characteristics. The same logic applies to local competition. Therefore, firms will make production and location decisions based on $c$. I call this combined index $c$ the *appeal* of a city.

Formally, the location choice problem (21) becomes:

$$c^*(z) = \underset{c}{\operatorname{argmax}} \ \log M(c) + \log \frac{\psi\left(\frac{\mathbb{C}(c)}{z}\right)\varphi\left(\psi\left(\frac{\mathbb{C}(c)}{z}\right)\right)}{\mathbb{P}(c)/\mathbb{D}(c)} - \log \frac{\mu\left(\frac{\mathbb{C}(c)}{z}\right)}{\mu\left(\frac{\mathbb{C}(c)}{z}\right) - 1} \tag{25}$$

The first-order condition of this problem reveals the main economic force that generates firm sorting in general equilibrium:[23]

[23] Formally, this condition anticipates that equilibrium conditions involve only continuously differentiable fixed point functionals.

$$\underbrace{\frac{\partial}{\partial c}\left(\log M(c) - \log \frac{\mathbb{P}(c)}{\mathbb{D}(c)}\right)}_{\substack{\Delta \text{ in sales from locating} \\ \text{in more appealing cities}}} = \underbrace{\frac{\mathbb{C}'(c)}{\mathbb{C}(c)} \frac{1}{\mu\left(\frac{\mathbb{C}(c)}{z}\right) - 1}}_{\substack{\Delta \text{ in fraction of sales going} \\ \text{to inputs of production}}} \tag{26}$$

The left-hand-side (LHS) of (26) encodes the benefits of locating in more appealing cities (high $c$ cities). First, through $M(c)$, it captures how the size of cities changes when they become more attractive for firms. Second, $\mathbb{P}(c)/\mathbb{D}(c)$ captures how market share changes as $c$ increases.[24] The sum of these two terms represents the percentual increase in sales from locating in high $c$ locations. Importantly, the LHS of (26) does not depend on the firm's productivity. Therefore, the net increase in sales from locating in more appealing cities is the same for all producers.

The right-hand-side (RHS) of (26) represents the costs of locating in more competitive cities and embeds the primary sorting mechanism this paper considers. This term reveals that the cost of locating in cities with high competition is lower for high-productivity firms. When firms locate in more competitive cities, they turn a lower fraction of their sales into profits because they charge lower markups. Nevertheless, the fraction of sales firms forgo locating in more competitive environments is smaller for more productive firms. Intuitively, because of the pricing complementarities, more productive firms charge higher markups and therefore have "more room" to absorb the higher competitive pressures in more competitive cities: the drop in markups does not affect them that much, as they initially had high margins. On the other hand, low-productivity firms are affected relatively more as they initially had lower margins.

Of course, the level competition level $\mathbb{C}(c)$ is an endogenous object determined in equilibrium. This object ultimately depends on the strength of local price and input competition. Whether more appealing cities are more competitive or not is determined in general equilibrium. As formalized in Proposition 1, more productive firms will self-select in more appealing cities that are bigger and where competition is endogenously tougher. The main driver of this spatial sorting is the pricing complementarities that make more productive firms gain relatively more from increased sales a bigger city allows.

The assignment problem (25) is a static assignment problem. However, it departs from traditional problems that consider sorting between two-sided markets with endogenously given characteristics.[25] The distinct feature of this problem is that complementarities arise between the productivity of the firm, $z$, and the endogenous competition index, $\mathbb{C}(c)$.[26] Local producers make their location decisions based on city competition levels. Their location decisions further determine local prices, wages, and commercial structure rents, which ultimately feed back to the competition index. This

[24] Appendix B.1 shows that, to a first-order, when comparing two locations, the firm price changes in each location do not affect its market share. That is, because firms are already choosing their optimal price, the envelope theorem implies that $\partial \log \psi\left(\frac{\mathbb{C}(c)}{z}\right) \varphi\left(\psi\left(\frac{\mathbb{C}(c)}{z}\right)\right) / \partial c = 0$.

[25] See Galichon (2016) for a complete exposition of this type of problems.

[26] Bilal (2023) also considers a framework in which firms sort based on an endogenous equilibrium object. He considers an assignment problem in which, due to labor market frictions, complementarities arise between the firms' productivity and the endogenous local vacancy meeting rate.

loop between the firms' location decision and the competition of a city differentiates the assignment problem from those considered in previous studies.

To characterize the problem (25), I define an *assignment pair* as a pair of functions $c \to (z(c), M(c))$, where $z(c)$ is the assignment function of local producers to cities, which is the inverse of $c^*(z)$. The function $M(c)$ is the equilibrium market size that supports this location choice.

**Proposition 1** (Sorting)**.** *Suppose that Assumption 1 holds, and that* $\xi(\overline{\sigma} - 1) > 1$*, where* $\xi$ *is given by:*

$$\xi \equiv 1 + \frac{\theta(\alpha + \eta\beta) + \gamma(1 + \theta(1 - \eta\beta))}{\gamma(1 + \phi)} \tag{27}$$

*Then, there exists a threshold* $\underline{\varepsilon}$ *such that for all* $\varepsilon \in (0, \underline{\varepsilon}]$ *there exists a unique solution to (25). In this solution, the functions* $z(c)$*,* $M(c)$*, and* $\mathbb{C}(c)$ *are strictly increasing.*

*Proof.* See Appendix B.1. □

Proposition 1 demonstrates the existence of the assignment between city appeal $c$ and local producer's productivity $z$ with positive assortative matching: more productive producers go to more appealing cities. Under this assignment, more appealing cities are bigger and have tougher competition. Intuitively, more appealing cities attract more workers and pay higher wages. This triggers the incentives for local producers to enter such markets. In turn, local competition increases. Because of pricing complementarities, only the most productive firms can sustain the high level of local competition. Low-productivity firms opt-out and locate in smaller cities where competition is slack.

The parameter restrictions in Proposition 1 ensure the existence of a unique assignment. The term $\xi$ captures different congestion forces in the framework. These forces prevent cities from having an infinite size in equilibrium.[27] When $\varepsilon$ is not too large, workers' valuation from an additional variety is given by $1/(\overline{\sigma} - 1)$, which is the central agglomeration force in this framework. Therefore, the condition $\xi(\overline{\sigma} - 1) > 1$ captures the standard condition for the uniqueness of equilibrium in general equilibrium spatial models: congestion forces need to be greater than agglomeration forces.[28]

## 2.6 Markups across Cities

Having characterized the location decision of the local varieties producers, we turn to study how these decisions shape the distribution of markups across cities. Under the assumptions of Proposition 1, local producers in $c$ charge a markup equal to $\mu\left(\mathbb{C}(c)/z(c)\right)$. The term $z(c)$ is the assignment function from Proposition 1 and reflects the productivity level of the local producers.

[27] In general, these forces capture the fact that as cities grow, the cost of labor and buildings (housing and commercial structures) grow as well. Moreover, they also capture that cities are not perfect substitutes for workers because of the idiosyncratic location tastes.

[28] See Redding and Rossi-Hansberg (2017) Section 3.5 for a detailed discussion on congestion and agglomeration forces in a canonical spatial quantitative model.

Let $\mathcal{M}(c)$ denote the city-level markup, implicitly defined by the city-level labor share in local goods:

$$\frac{W(c)L^N(c)}{M(c)} = \frac{\beta}{\mathcal{M}(c)}.$$

Combining this definition with the firm-level labor share (19), one can show that the city-level markup is a sales-weighted harmonic average of the firm-level markups.[29] Nevertheless, under Proposition 1 all firms in $c$ charge the same markup and therefore the aggregate markup $\mathcal{M}(c)$ is:

$$\mathcal{M}(c) = \mu\left(\frac{\mathbb{C}(c)}{z(c)}\right) \tag{28}$$

Equation (28) reveals two opposite forces that determine the city-level markup. On the one hand, bigger cities have tougher competition: $\mathbb{C}(c)$ is strictly increasing. Therefore, a *competition force* in bigger cities pushes the city-level markup down. On the other hand, bigger cities attract more productive firms: $z(c)$ is strictly increasing. Hence, a *selection force* in bigger cities pushes the level of markups up. The relative strength of these two forces determines if bigger cities have higher or lower markups in equilibrium.

To facilitate the characterization of the level of markups across cities, I adopt a particular functional form of the common productivity distribution of local producers $G(\cdot)$.

**Assumption 2** (Firm Productivity Distribution)**.** *The common productivity distribution of local producers is a truncated Pareto with support* $[z_L, z_H]$ *and shape parameter* $\delta$*:*

$$G(z) = \frac{1 - \left(\frac{z_L}{z}\right)^{\delta}}{1 - \left(\frac{z_L}{z_H}\right)^{\delta}}, \qquad \delta > 0. \tag{29}$$

**Proposition 2** (Markups and City Size)**.** *Suppose that Assumptions 1 and 2 hold. There exists a threshold* $\underline{\delta}$ *such that*

1. *If* $1/\delta < 1/\underline{\delta}$*,* $\mathcal{M}(c)$ *is strictly decreasing.*
2. *If* $1/\delta > 1/\underline{\delta}$*,* $\mathcal{M}(c)$ *is strictly increasing.*

*The threshold* $\underline{\delta}$ *depends on the demand parameters* $\overline{\sigma}$*,* $\varepsilon$*, and on the distribution of cities appeal ,* $F(c)$*.*

*Proof.* See Appendix B.2. □

Proposition 2 establishes that the productivity dispersion of local producers determines markups across cities. The inverse of the productivity distribution shape parameter, $1/\delta$, is a measure of how dispersed is the productivity of local producers. When the productivity of local producers

[29] Edmond, Midrigan, and Xu (2023) obtain an equivalent expression when defining the sector-level markup.

is not that dispersed, relative to the dispersion of exogenous characteristics of cities, bigger cities have lower markups: the competition force dominates. If, on the other hand, local producers differ too much in their productivity relative to how cities differ in their exogenous characteristics, then bigger cities have higher markups: the selection force dominates.

The results in Proposition 2 resemble recent findings in the literature of endogenous variable markups. Recall that, under Proposition 1, bigger cities are more competitive. However, depending on local producers' productivity dispersion, bigger cities can have higher or lower markups. Therefore, the level of markups in a given city should not be taken as *prima-facie* evidence of reduced competition. Baqaee, Farhi, and Sangani (2023) and Matsuyama and Ushchev (2022) also find conditions under which larger markets could have higher markups.

The proof in B.2 provides intuition on how the results in 2 extend to a more general local producers productivity distribution.

## 2.7 Equilibrium

Having characterized how the location of local varieties producers determines spatial differences in market power, I close the model and set the conditions for the decentralized equilibrium.

First, the land developer's production function leads to an equilibrium buildings supply equal to $R(c)^{\phi}$ and traded good demand of $\phi R(c)^{1+\phi}/(1+\phi)$. Then, local housing and local labor markets clear in every city:

$$R(c)^{\phi} = L(c)H(c) + S^N(c) + S^T(c), \qquad L(c) = L^N(c) + L^T(c), \tag{30}$$

where $H(c)$ is given by (6), $L(c)$ is given by (11), $L^N(c)$ and $S^N(c)$ are given by (19), and $L^T(c)$ and $S^T(c)$ are given by (23). Moreover, the traded good market must also clear. Recall that the traded good is consumed by workers, used to build housing and structures, and used to pay the local producers' entry costs. Hence, the traded good market clearing condition is:

$$\int_c Q^T(c)dF(c) = \int_c \left( L(c)Q(c) + \frac{\phi}{1+\phi} R(c)^{1+\phi} \right) dF(c) + c_e M_e \tag{31}$$

Because of free entry, local producers' expected profits must equal the entry cost. Furthermore, the labor market clears in the aggregate:

$$\int_c \Pi(z, c^*(z))dG(z) = c_e \qquad \int_c L(c)dF(c) = \overline{L} \tag{32}$$

Proposition 2 guarantees that there exists a unique equilibrium in this economy.

**Proposition 3** (Existence and uniqueness)**.** *Suppose the assumptions of Proposition 1 hold, and that the supports of $F(c)$ and $G_z(z)$ are not too large. There exists a unique decentralized equilibrium. This equilibrium exhibits positive assortative matching.*

*Proof.* See Appendix B.3. □

Proposition 3 finishes the positive analysis of the theoretical framework. Next section explores the model's normative implications.

# 3 Efficiency

In a single location model with variable markups, there are two different margins of inefficiency. As pointed out by Baqaee, Farhi, and Sangani (2023) and Edmond, Midrigan, and Xu (2023), variable markups can lead to inefficient overall entry and misallocation of factors of productions across firms.[30] The entry inefficiency arises because profits (private return) from the marginal entrant differ from the consumer surplus their entry generates (social return). Moreover, misallocation of factors of production arises when more productive firms charge higher markups.[31] Relative to the social optimum, more productive firms are too small, and aggregate welfare could increase by reallocating production from low to high-productivity firms.

With geography, local good producers make another decision: choose where to locate. With this additional layer, the overall entry margin may be inefficient, and the city-specific entry rate could be inefficient. The equilibrium allocation in the decentralized equilibrium is inefficient because of two opposite externalities that arise with local entry.

Firms create a positive externality when entering a particular city. Because consumer values variety in local goods, when a firm enters a city, it raises consumer surplus by creating a new good. I call this externality, *variety gains externality*. Nevertheless, firms can only partially appropriate the gain in consumer surplus into their profits. This non-appropriability reduces firms' incentives to enter a particular city, leading to insufficient entry. From a social planner's perspective, we would like to have more firms in particular locations.

Firms also create a negative externality when entering a city. Because local varieties are imperfect substitutes, when a producer enters a city, it reduces the consumption of the existing varieties. Thus, firms impose a negative externality on incumbents by reducing their profits. This is a *business stealing* externality. There is excessive entry because firms do not internalize their effect on other producers. From a social planner perspective, we would like to have fewer varieties in a particular location and increase the consumption of the existing ones.

It is worth to highlight that the variety gains and the business stealing externality are not specific to my framework and are present in standard models of firm entry.[32] Nevertheless, in the commonly used models with CES preferences, these two externalities are always constant and offset each other (Matsuyama and Ushchev (2021)).

In equilibrium, whether there is too much or too little entry in a particular city depends on the strength of the variety of gains and business stealing externalities. In the spatial equilibrium

[30] Moreover, with overhead costs, there is a third margin of inefficiency: the selection cutoff in productivity.
[31] Which is the case when Marshall's second law holds.
[32] This was early pointed out by Mankiw and Whinston (1986)

described in the previous section, the variety gains externality is higher in small cities, and the business stealing externality is higher in bigger cities. Consequently, there is too much entry in bigger cities and too little in small cities. The spatial sorting of firms through pricing complementarities leads to more productive firms over-concentrating in larger markets relative to the social optimum. This inefficiency ignites a "top-down" effect on other cities, generating misallocation throughout the economy.

To better understand the spatial nature of the variety gains and the business stealing externalities, consider two locations $c_1 < c_2$. Because location $c_2$ is more appealing, it is bigger and more competitive, $M(c_1) < M(c_2)$ and $C(c_1) < C(c_2)$. A lower competition level in the small city reflects that it cannot attract too many local producers, and the ones that decide to operate there are of low productivity, $z(c_1) < z(c_2)$. Therefore, consumers in the small location benefit more from an additional variety than consumers in the big city. On the other hand, bigger cities can attract the more productive firms because potential profits are higher relative to small cities.[33] Therefore, the incumbents' profit loss from a marginal entrant is higher in big cities than in smaller cities. As a result, the variety gains externality dominates in smaller markets, while the business stealing externality dominates in bigger ones.

## 3.1 Social Planner's Problem

I formalize the previous arguments by characterizing the planner's problem. An utilitarian planner maximizes the population-weighted sum of workers' utility in every city. The planner chooses the location of local producers and population subject to workers' idiosyncratic location tastes. The planner also chooses the allocation of labor into traded good production and local varieties production in every city. Moreover, she is subject to the housing supply technology in every location. I relegate the formal definition of the planning problem to Appendix D and characterize the solution in the main text. I use $SP$ superscripts for the solutions in the planner problem and $DE$ superscripts for the decentralized equilibrium. The decentralized equilibrium is inefficient when the decentralized allocation does not coincide with the planning one.

**Proposition 4** (Efficient Allocation)**.** *The decentralized equilibrium is inefficient. Moreover, suppose that the supports of $G(z)$ and $F(c)$ are not too large as in Proposition 3. Then, for all $c$:*

$$z^{SP}(c) > z^{DE}(c) \qquad \textit{for all} \qquad c \in (c_L, c_H). \tag{33}$$

*Proof.* See Appendix D.1. □

Proposition 4 establishes that the decentralized equilibrium is inefficient. In the baseline framework, markups generate inefficiencies through three channels. First, they distort the relative consumption between local varieties, housing, and the traded good. Second, they distort the location decisions of firms. Third, they distort the aggregate entry margin of local producers. Importantly,

[33] Note that if profits were higher in smaller cities, then high productivity firms would have a profitable deviation by locating in smaller cities, which contradicts the results of Proposition 1.

because local producers in each city have the same productivity, there is no misallocation within a city in the sense of Hsieh and Klenow (2009).

Equation (33) shows that firms are misallocated across cities. Local producers in the decentralized equilibrium are not productive enough relative to the social planners' solution. More productive firms are too concentrated in bigger cities in the decentralized equilibrium. On the other hand, for any city $c$, the social planner selects more productive firms $z^{DE}(c) < z^{SP}(c)$. Therefore, there is a misallocation of firms across cities: aggregate welfare can increase by reallocating productive producers from big to small cities.

Whether big cities have higher or smaller markups affects firm misallocation. As Appendix D.2 formalizes, the slope of markups across cities exacerbates the business stealing externality. To understand this mechanism, it is helpful to consider two economies: one in which markups in big cities are low and another in which markups are high. In the first economy, more productive firms face the trade-off between locating in larger markets when they sell more but make lower margins. This trade-off reduces the incentives for setting production in big cities, and marginal producers find it optimal to reallocate to smaller locations. On the other hand, in the second economy, where markups are high in bigger cities, firms no longer face this trade-off: they sell more and have larger margins in such locations. Of course, low-productive producers still self-select into smaller cities because of the pricing complementarities. However, the more productive producers who can handle the high competitive pressure in bigger cities will over-concentrate even more in such locations than in the first economy scenario.

As the results in the empirical section highlight, the slope of the relationship between markups and city size is then informative of the degree of misallocation in the economy. Even though firms always over-concentrate in big cities, a negative slope suggests that misallocation across space is less severe than in a situation with a positive slope.

## 3.2 First-best Implementation

How can efficiency in the decentralized equilibrium be restored? Proposition 5 shows that the first-best allocation can be attained by implementing a location-specific subsidy per unit sold. This subsidy corrects the three margins of inefficiency previously discussed: markups, misallocation of firms across cities, and overall entry. Finally, the subsidy is financed by a flat labor tax.[34]

Formally, consider $T(y,c)$ that is city-specific and depends the quantities sold, $y$:

$$T(y,c) = \left[ \underbrace{\Upsilon\left(\frac{y}{Y(c)}\right)}_{\text{worker's utility}} - \underbrace{\Upsilon'\left(\frac{y}{Y(c)}\right)\frac{y}{Y(c)}}_{\text{original revenue curve}} \right] \frac{\mathbb{D}(c)}{\mathbb{P}(c)} M(c) \tag{34}$$

The policy in (34) affects firm revenue in two ways. First, it takes away the sales from the firm's original revenue curve: the term corresponding to $\Upsilon'\left(y/Y(c)\right)\left(y/Y(c)\right)$. Second, it returns

[34] That is, workers earnings in every city are equal to $(1-\tau)W(c)$, where $\tau$ is the tax.

revenues proportionately to $\Upsilon\left(y/Y(c)\right)$, which measures the relative "utility" each firm generates. Under this policy, the profits for firm $z$ in city $c$ are then given by:

$$\Pi(z,c) = \left[\Upsilon\left(\frac{y(z,c)}{Y(c)}\right) - \frac{\mathbb{C}(c)}{z}\frac{y(z,c)}{Y(c)}\right]\frac{\mathbb{D}(c)}{\mathbb{P}(c)}M(c) \tag{35}$$

Equation (35) reveals that the transfer eliminates any incentives for firms to charge a markup. When sales come from the original revenue curve, producers are incentivized to shrink production to maximize sales. However, when sales come proportional to $\Upsilon\left(y/Y(c)\right)$, firms have the incentives to maximize the units produced. In turn, firms will produce at marginal cost and exert no market power. As Proposition 5 clarifies, when firms profits are given by (35), firm misallocation across cities is also eliminated.

**Proposition 5** (Optimal Policy)**.** *Under the location-specific subsidy* (34)*, the decentralized equilibrium allocation coincides with the planner solution.*

*Proof.* See Appendix D.3. □

Proposition 5 shows that the subsidy (35) corrects the three margins of inefficiency in the decentralized equilibrium. First, it eliminates markups, which corrects the inefficient relative consumption between the bundle of local varieties, housing, and the traded good. Second, it gives the right incentives for firms to locate efficiently. Lastly, it also corrects the overall entry into the economy. As Appendix D.3 shows, in equilibrium, firms profits (35) can be written as:

$$\Pi(z,c) = \Big[\underbrace{\delta\left(\frac{y(z,c)}{Y(c)}\right)}_{\text{consumer surplus}} - 1\Big]\underbrace{\Upsilon\left(\frac{y(z,c)}{Y(c)}\right)}_{\text{market share}}M(c), \tag{36}$$

where $\delta\left(\frac{y(z,c)}{Y(c)}\right)$ is the ratio of the consumer surplus to firm sales.[35] In equilibrium, firms capture a share of the total revenue in a market proportionally to workers' utility. Moreover, firms' profits exactly coincide with the consumer surplus they generate. Then, because firms are now correctly compensated for their effect on workers' utility, the location and the overall entry margin are corrected. Finally, it is worth to highlight that (34) generalizes the insights of Edmond, Midrigan, and Xu (2023). In their setting, the optimal policy for a single market is similar to (35).

This section outlined a spatial general equilibrium model in which spatial markup differences arise because of the location choice of heterogeneous local producers. The framework highlights that differences in markups across are explained by differences in local competition and the productivity of local producers. Moreover, In the next section, I study the framework's predictions using data from local producers in the United States.

[35] Formally, $\delta\left(\frac{y(z,c)}{Y(c)}\right) = \Upsilon\left(\frac{y(z,c)}{Y(c)}\right) / \left(\frac{y(z,c)}{Y(c)}\Upsilon'\left(\frac{y(z,c)}{Y(c)}\right)\right)$. See Figure 1 in Baqaee, Farhi, and Sangani (2023) for a visual representation of this object.

# 4 Empirical Analysis

In this section, I empirically investigate the theory predictions. When taking the model to the data, we need to take a stand of a definition of a city and on the map between locations in the model (continuum) and cities in the data (discrete). In turn, I define a city in the data as a county and I consider a county as being a collection of related locations in the model. Formally, a county is an interval $[c, c + dc]$ of cities in the model.

To conduct the empirical investigation, I first describe the U.S. establishment-level data used for all exercises. Second, I introduce the classification of traded and local (non-traded) sectors I use through the empirical exercises. Then, I perform model validation exercises. Finally, I outline and implement the empirical strategy to estimate markups and study the variation across U.S. cities.

## 4.1 Data

The primary dataset used in this project is the micro-data from the U.S. Census Longitudinal Business Database (LBD). This data source uses administrative employment records of every non-farm private establishment in the U.S. economy. The establishment-level variables I used are employment, wage bill, geographic location (county), industry (6-digit NAICS), and the establishment identifier.

I supplement the LBD data with sales data at the establishment level from the Economic Censuses every five years from 2002 to 2017. Specifically, I use the micro-data from the Census of Construction Industries, Manufacturing, Retail Trade, Census of Services, Wholesale Trade, Finance, Insurance and Real Estate, and the Census of Transportation, Communications and Utilities. I use the establishment identifier to link the establishment in the Economic Censuses to the establishments in the LBD. The final sample is the establishments in the LBD with matched sales data from the Economic Censuses.[36] I use 2017 as the baseline year, leaving 2002, 2007, and 2012 for robustness exercises.

I use the Census of Manufactures to perform additional markup estimation exercises. The Census of Manufactures has detailed data on establishment materials, capital (equipment and structures), and energy expenditures. Unfortunately, such detailed data is not available in the other Economic Censuses. I construct real capital, materials, and labor measures using standard procedures used in the productivity estimation literature (see Foster, Grim, and Haltiwanger (2016)).

In the baseline exercises, I associate a city with a county. Focusing on continental U.S., I include 3080 counties in my estimation. For some the robustness exercises, I define cities as Commuting Zones.[37]

[36] This is virtually the same sample used in Hsieh and Rossi-Hansberg (2023).

[37] To map counties to commuting zones, I use the crosswalk provided by Autor and Dorn (2013).

Table 1: Summary statistics baseline 2017 sample

| | All Industries (1) | Local Industries (2) | Traded Industries (3) |
|---|---|---|---|
| Number of establishments | 6,655,000 | 5,075,000 | 1,579,000 |
| Avg. Employment (# of workers) | 17.82 | 15.70 | 24.61 |
| Avg. Sales (thousands) | 3,463 | 2,376 | 6,955 |
| Avg. Wage bill (thousands) | 627.4 | 488.1 | 1,075 |
| Agg. employment share | ... | 0.67 | 0.33 |
| Agg. sales share | ... | 0.52 | 0.48 |
| Agg. wage bill share | ... | 0.59 | 0.41 |

**Notes:** Table 1 displays summary statistics for the 2017 LBD-EC matched sample (baseline sample). The traded-local industries classification is based on Delgado, Porter, and Stern (2015).

## 4.2 Local Industries

I use the definition of Delgado, Porter, and Stern (2015) to classify establishments in the LBD as traded or local producers. Broadly, this definition classifies 6-digit NAICS industries into "Traded" or "Local" based on employment specialization, geographic concentration, and distance to final consumers. Local producers belong to industries in most of the geographic areas and sell to local consumers. On the other hand, traded industries sell to other regions and are sometimes geographically concentrated. Formally, the authors group 310 6-digit NAICS industries as Local and 778 6-digit NAICS industries as Traded.[38] Using the industry codes from the LBD, I classify an establishment as a local producer if it belongs to any of the 310 local industries.[39]

Table 1 displays summary statistics for the baseline sample. The sample includes 85% of all the LBD establishments in 2017. Establishments operating in local industries are to be smaller in number of workers, sales, and wage bill compared to their counterparts in traded industries. Nevertheless, the number of local establishments is almost three times that of traded ones. Hence, in the aggregate, local industries represent more than half of the U.S. economic activity by employing 67% of the labor force and by accounting for 52% of the total sales and 59% of total labor income.

## 4.3 Model Validation

Before analyzing the empirical patterns of markups across U.S. cities, I provide empirical support for the model's sorting prediction. Proposition 1 indicates that local producers are more productive in bigger cities. Because firm productivity is not observed in the data, I consider how two proxies of firm productivity relate to city size.

**Labor productivity and city size.** The first proxy for firm productivity is labor productivity.

[38] For the full list of 310 local NAICS industries see Cluster Mapping Project.
[39] This classification is used in Berger, Herkenhoff, and Mongey (2022).

Ideally, one would like to study output (physical quantities) per worker. However, I cannot separate prices and quantities as I only observe sales at the establishment level. Therefore, I define labor productivity at the establishment level as sales per worker.

I construct measures of labor productivity and size for every county. Using the establishment's location and local-traded classification, I compute sales per worker for establishments in local industries across all counties. Then, for every county, I compute the average sales per worker across all establishments in local industries in a given county. This is the measure of labor productivity at the county level. On the other hand, guided by the theoretical framework, I define the "size" of a county as the total income of workers residing in that county.[40]

Figure 1(a) displays a bin-scatter of counties' log labor productivity and log county size. As Proposition 1 establishes, county size and county labor productivity have a positive and significant relationship. An increase of 1% in a county's size is associated with a 0.03% increase in county labor productivity. Moreover, counties in the top decile of the county-size distribution have a labor productivity 13% higher than counties at the bottom.

**Establishment size and county size.** The second firm productivity proxy we consider is establishment size. Appendix A.2 shows that the model predicts that producers in bigger cities employ more workers than producers in smaller cities. To empirically investigate this prediction, I first measure establishment size in each county by computing the average establishment employment for local industries. Then, I split counties into 100 equally-sized bins according to their total labor income and compute the average establishment size across counties in each bin. Generally, we compute the average establishment size for counties across the percentiles of the county-size distribution.

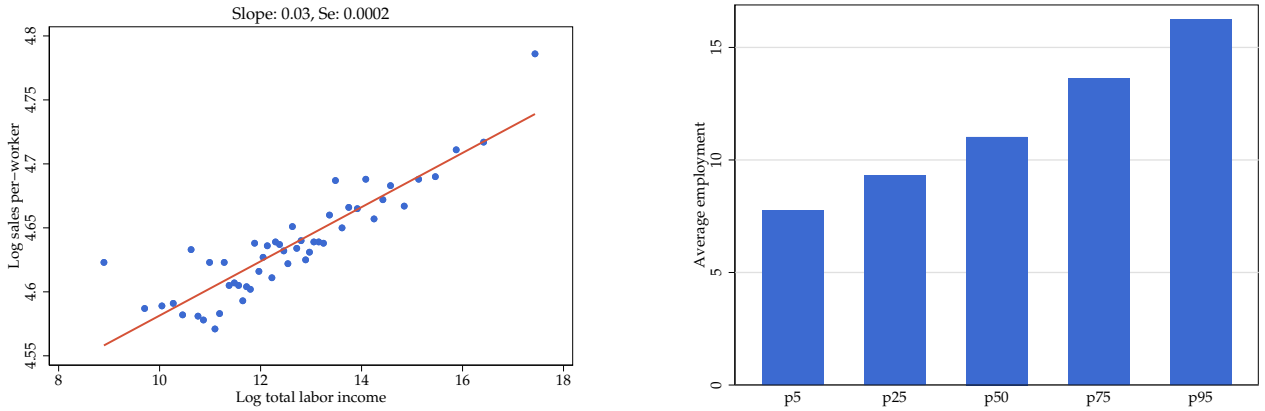
Figure 1(b) shows the average employment for local establishments in different counties across the county-size distribution. In particular, it displays the average employment for local establishments in counties in the 5th, 25th, 50th, 75th, and 95th percentiles of the city-size distribution. A typical local establishment in the smaller counties (5th percentile) has 7.76 workers; in the largest counties (95th percentile), it employs 16.23 workers. This demonstrates that average employment in local industries doubles across the county-size distribution.

## 4.4 Markup Estimation

This section outlines the empirical strategy to estimate markups for establishments in local industries. Using the insights of De Loecker (2011), I develop an alternative method for markup estimation that combines consumer preferences with the firm optimal input decisions. Before proceeding with the description of the method, I discuss the limitations of applying some of the existing methods in my LBD-Economic Census sample.

[40] As my model indicates, I consider all workers regardless of whether they are employed in local or traded industries.

Figure 1: Local industries revenue per worker and average employment across counties




(a) Sales per worker and total labor income

(b) Average estab. employment across counties.

**Notes:** Figure 1(a) shows a bin-scatter of county log average sales per worker and log total labor income for local industries. The bin-scatter considers 50 equally sized county bins according to their total labor income. Average sales per-worker is computed among establishments in local industries. Figure 1(b) displays average employment for establishments in local industries across counties in different percentiles of the county-size distribution. Different bars indicate percentiles of the county-size distribution: 5th, 25th, 50th, 75th, and 95th percentiles. The height of each bar represents the average employment of establishments in local industries across counties in each percentile. County size is defined as total labor income.

### 4.4.1 Existing Methods

I build on the production approach to estimate markups. Originally developed by Hall (1988) and recently extended by De Loecker and Warzynski (2012), this approach produces markup estimates using data on sales, variable input expenditures, paired with estimates of output elasticities. In contrast, an alternative procedure often refereed as the demand approach, uses data on prices and quantities to estimate the marginal cost of production. With estimates on own and cross-price elasticities across goods, markups can be recovered from the firms pricing first-order conditions after specifying the market structure under which firms compete.[41] I do not observe prices or product characteristics in the LBD or the EC's data, and therefore I cannot implement the demand approach.

I index establishments by $j$ and counties by $c$ in the data. Under certain regularity conditions of the firms' cost-minimization problem, markup for establishment $j$ in county $c$ can be expressed as the ratio of the output elasticity of a flexible input and the cost-shares of sales of that input.[42] The conditions for the ratio estimator hold in the setting outlined in Section 2 and equation (19) implies that:

$$\mu_{jc} = \frac{\beta}{\alpha^{l}_{jc}}, \tag{37}$$

[41] See Ackerberg et al. (2007) and Berry, Gaynor, and Scott Morton (2019) for excellent overviews.

[42] A flexible input is one that: 1) can be adjusted freely every period, and 2) establishments take as given the price of the input. The latter condition rules out the possibility of monopsony power that inputs the market.

where $\alpha_{jc}^{l} = (W_c l_{jc})/(p_{jc} y_{jc})$ is the labor expenditure share of total sales, which is observed in the data. Then, using (37), one can form an estimate of the markup by obtaining an estimate of the labor output elasticity, $\hat{\mu}_{jc} = \hat{\beta}/\alpha_{jc}^{l}$. This estimator is commonly called the "ratio estimator".[43] Under this approach, the main econometric challenge is to estimate production elasticities.

The first procedure to estimate output elasticities is the production function approach. Under this alternative, researchers estimate a production function by regressing output on inputs. The estimation is usually done by implementing a control function approach as in Olley and Pakes (1996), Levinsohn and Petrin (2003), Ackerberg, Caves, and Frazer (2015), and Gandhi, Navarro, and Rivers (2020), or by estimating dynamic panel models as in Arellano and Bover (1995) and Blundell and Bond (1998, 2000). Each of these approaches has costs and benefits. However, one common requirement is data on physical quantities as the output measure.[44] Unobserved output price differences confound the identification of the production function parameters when sales are used as an output measure.[45] Data on physical quantities for establishments throughout different economic sectors in the U.S. does not exist.[46] In particular, I observe sales as the output measure in the LBD-EC sample. Therefore, we cannot implement the production function approach to form the ratio estimator.

The second alternative to estimate production elasticities is the cost-share approach.[47] Relying on cost minimization conditions, an input's output elasticity equals the input's cost share of total costs times the scale elasticity.[48] In contrast to the production function alternative, this approach does not require data on physical quantities. However, it requires data on the establishment's total costs. As highlighted by De Loecker and Syverson (2021), data on total costs is rare, with capital costs being the most difficult to observe. Indeed, data on the U.S. establishment's total costs does not exist except for publicly traded companies and manufacturing establishments. Thus, I cannot implement the cost-share approach to the LBD-EC sample to estimate markups through the ratio estimator.

In sum, data limitations prevent the implementation of the ratio estimator. As an alternative, in the same spirit of De Loecker (2011), I use the demand structure from my model to overcome the identification challenge. De Loecker (2011) uses a CES demand structure to control for unobserved prices in a production function estimation context. Similarly, I use the demand structure from the theory section to construct a markup control function.[49] The following section describes in detail this alternative procedure.

[43] See De Loecker and Warzynski (2012) for a detailed derivation of this estimator.

[44] De Loecker and Syverson (2021) offer an exhaustive review of the control functions and dynamic panel models.

[45] Bond et al. (2021) discusses pitfalls of using the ratio estimator without data on physical quantities.

[46] Exemptions are Manufacturing sub-samples in Foster, Haltiwanger, and Syverson (2008) and Atalay (2014).

[47] De Loecker, Eeckhout, and Unger (2020) and Edmond, Midrigan, and Xu (2023) use this approach to estimate markups for publicly traded firms and manufacturing establishments, respectively.

[48] The scale elasticity is the degree of returns to scale of the production technology.

[49] The main difference between my framework and De Loecker (2011) is that my demand structure allows for variable markups.

### 4.4.2 Alternative Procedure

To avoid estimates dependency on functional forms specifications, I consider a general parametrization of the Kimball aggregator $\Upsilon(\cdot)$ with the only assumption of a choke price.[50]

**Assumption 3** (Kimball Aggregator for Markup Estimation)**.** *Assume $\Upsilon(\cdot)$ is a strictly increasing and concave function satisfying $\Upsilon(0) = 0$. Moreover, assume there exists $\overline{p} < \infty$ such that:*

$$\left(\Upsilon'\right)^{-1}(\overline{p}) = 0. \tag{38}$$

Recall from (7) that workers relative demand is given by the inverse of the derivative of the Kimball aggregator.[51] Hence, condition (38) implies the intuitive idea that a finite price exists at which workers demand zero quantities.

On the other hand, we can re-organize (37) and take logs to obtain:

$$\log \alpha_{jc}^{l} = \beta - \log \mu_{jc} \tag{39}$$

Note that the LHS of (39) is observed in the data. Therefore, one could potentially estimate markups as the residual of a regression of log labor cost share of revenue and a constant. Nevertheless, this procedure has two potential threats. First, any measurement error on the labor cost share of revenue will be absorbed in the error term confounding markup estimates. Second, as Appendix E.3 illustrates, if one considers a more general production function in which output elasticities are not constant and vary with inputs, the potential correlation between markups and input usage invalidates the identification of markups as residuals from (39). I use the demand system in Assumption 3 to construct a markup control function to avoid these issues.

Let $p_{jc}$ be the price establishment $j$ charges in county $c$. Moreover, let $D_c$ and $P_c$ be the competition and ideal price indices in county $c$, respectively. Using the Lerner formula, we can $\mu_{jc}$ as a function on the relative price:

$$\begin{aligned} \mu_{jc} &= \mu\left(\frac{p_{jc}}{D_c}\right), \\ &= \frac{1}{1 - \frac{1}{\sigma\left(\frac{p_{jc}}{D_c}\right)}} \end{aligned} \tag{40}$$

where $\sigma(\cdot)$ is given by (14). Because there is no price information in the U.S. micro-data, the object $p_{jc}/D_c$ is unobserved. However, we can use the demand system to express relative prices

[50] The Klenow and Willis (2016) introduced in Assumption 1 has a choke price. The CES functional form for $\Upsilon(\cdot)$ has no choke price.

[51] The concavity of $\Upsilon(\cdot)$ guarantees the existence of this inverse function.

as a function of sales market shares. Let $s_{jc}$ be th sales share of establishment $j$ in county $c$.[52] Appendix E.1 shows that we can express the sales share as:

$$s_{jc} = \frac{D_c}{P_c}\Upsilon'\left(\frac{p_{jc}}{D_c}\right)\frac{p_{jc}}{D_c} \tag{41}$$

Appendix E.1 further shows that the function $\Upsilon'(x)x$ is strictly increasing, and therefore we can use (41) to solve for $p_{jc}/D_c$ as a function of the sales market share and the price index ratio $P_c/D_c$:

$$\frac{p_{jc}}{D_c} = \zeta\left(s_{jc}\frac{P_c}{D_c}\right), \tag{42}$$

where $\xi(x)$ is the inverse of the function $\Upsilon'(x)x$. By combining (40) and (42) we can write $\mu_{jc}$ as function of $s_{jc} \times (P_c/D_c)$:

$$\mu_{jc} = \mu\left(\zeta\left(s_{jc}\frac{P_c}{D_c}\right)\right) \tag{43}$$

The exact functional form of the markup function $\mu \circ \zeta$ depends on the parametrization of $\Upsilon$. However, to maintain the estimation parsimoniously, I use a semi-parametric approximation for the markup function and use a sieve series estimator as analyzed in Chen (2007) and used in the production function estimation context by Gandhi, Navarro, and Rivers (2020). Formally, I approximate the log markup function by a third-order degree polynomial in $s_{jc} \times (P_c/D_c)$:

$$\log\mu\left(\zeta\left(s_{jc}\frac{P_c}{D_c}\right)\right) = \varsigma_1 s_{jc}\frac{P_c}{D_c} + \varsigma_2\left(s_{jc}\frac{P_c}{D_c}\right)^2 + \varsigma_3\left(s_{jc}\frac{P_c}{D_c}\right)^3 + \upsilon_{jc}, \tag{44}$$

where $\upsilon_{jc}$ is an approximation error that goes to zero once one considers higher polynomial terms. Crucially, the approximation in (44) does not have a constant term. As shown in Appendix E.1, Assumption 3 implies that when producers have a zero sales share, they charge a markup equal to one. Intuitively, because of the choke price, firms with zero sales share face an infinite elasticity of demand and, therefore, have no markups. Combining (39) and (44) yields the equation from which markups are identified:

$$\log\alpha^l_{jc} = \beta - \underbrace{\varsigma_{1,c}s_{jc} - \varsigma_{2,c}s^2_{jc} - \varsigma_{3,c}s^3_{jc}}_{\equiv\log\mu_{jc}} - \upsilon_{jc}, \tag{45}$$

where $\varsigma_{1,c} \equiv \varsigma_1(P_c/D_c)$, $\varsigma_{2,c} \equiv \varsigma_2(P_c/D_c)^2$, and $\varsigma_{3,c} \equiv \varsigma_3(P_c/D_c)^3$. Because I do not observe the county price indices, I treat them as county fixed-effects, and hence (45) takes the form of a heterogeneous slopes model.

[52] Formally, $s_{jc} \equiv p_{jc}y_{jc}/\left(\sum_{j'\in c} p_{j'c}y_{j'c}\right)$.

Equation (45) reveals the variation that identifies markups. Conditional on the establishment technology, $\beta$, markups are identified using within-county variation in the sales shares and the labor cost share of sales. Within a county, establishments with high sales shares and low labor expenditure share have higher markups. The choke price allows us to separate the markup approximation function's constant (zero) from the labor output elasticity $\beta$. Thus, the levels of markup are correctly identified.[53]

Appendix E.2 shows how to extend the estimation when considering multiple sectors and controlling for potential labor market power.[54] In particular, when estimating markups by sector, the estimating equation takes the form of:

$$\log \alpha^{l}_{jnc} = \beta_n - \varsigma_{1,nc} s_{jnc} - \varsigma_{2,nc} s^2_{jnc} - \varsigma_{3,nc} s^3_{jnc} - \upsilon_{jnc}, \tag{46}$$

where $n$ index sector. In contrast to (45), the sector estimation uses the within-county-sector variation in sales share and labor cost share of sales to recover markups.

### 4.4.3 Markups for Establishments in Local Industries

This section presents the results for the markup estimation outlined in Section 4.4.2. Table 2 presents summary statistics for establishments in different local industries. The first row displays the results from the estimation for all local industries in (45). The remaining rows present markup estimates for the sector estimation in (46). For the sector estimation, I define a sector as a 2-digit NAICS industry.

There is considerable heterogeneity in markups across establishments in local industries. The median establishment in local industries has a markup of 1.43, which is the lines of the findings of De Loecker, Eeckhout, and Unger (2020) for publicly traded companies. However, I find significant cross-sectional heterogeneity, with establishments in the top decile of the markup distribution charging a markup seven times higher than establishments in the bottom decile.

There is also significant markup heterogeneity across local industries. On the one hand, the median and the 10th markup percentile are similar across different local sectors. Nonetheless, sectors like Manufacturing, Wholesale, and Information exhibit a mean markup significantly higher than the other sectors. These sectors also exhibit a larger p90 - p10 gap than the others.

### 4.4.4 Local Industries Markups across Cities

I now turn to the main empirical analysis of this section: markups across cities. I construct the sales-weighted harmonic mean of establishment markups to compute the county-level markup. Formally, following the implications of the theory in Section 2, the county aggregate markup $\mathcal{M}_c$ is defined as:

[53] This is not the case in similar approaches like Peters (2020).

[54] In the baseline estimations, I control for potential labor market power by adding a flexible polynomial in the establishments' wage bill share to (45).

Table 2: Summary Statistics: Markups for Establishments in Local Industries

| | Mean (1) | Median (2) | p10 (3) | p90 (4) |
|---|---|---|---|---|
| All Local | 2.57 | 1.43 | 1.07 | 7.78 |
| Local Construction | 1.79 | 1.13 | 1.01 | 3.25 |
| Local Manufacturing | 2.56 | 1.43 | 1.07 | 7.77 |
| Local Wholesale | 3.28 | 1.79 | 1.05 | 7.76 |
| Local Retail | 2.4 | 1.35 | 1.04 | 7.78 |
| Local Transportation and Warehousing | 1.89 | 1.16 | 1.02 | 3.82 |
| Local Information | 2.7 | 1.41 | 1.05 | 7.75 |
| Local Finance and Insurance | 1.67 | 1.17 | 1.03 | 2.75 |
| Local Real Estate | 1.29 | 1.1 | 1.02 | 1.85 |
| Local Profesional, Scientific and Technical Services | 1.24 | 1.08 | 1.01 | 1.69 |
| Local Administrative | 1.41 | 1.1 | 1.01 | 2.22 |
| Local Education Services | 1.11 | 1.06 | 1.01 | 1.3 |
| Local Healthcare | 1.54 | 1.14 | 1.02 | 2.48 |
| Local Arts, Entertainment, and Recreation | 1.36 | 1.13 | 1.02 | 2.07 |
| Local Accommodation and Food Services | 1.32 | 1.17 | 1.03 | 1.87 |
| Local Other Services | 1.18 | 1.08 | 1.01 | 1.52 |

**Notes:** Table 2 displays summary statistics for the estimated markups using (45). The first row considers all local establishments. The following rows display statistics for establishments in local industries for 2-digits NAICS sectors. Columns p10 and p90 denote the 10th and 90th percentile of the markup distribution, respectively. The traded-local industries classification is based on Delgado, Porter, and Stern (2015).

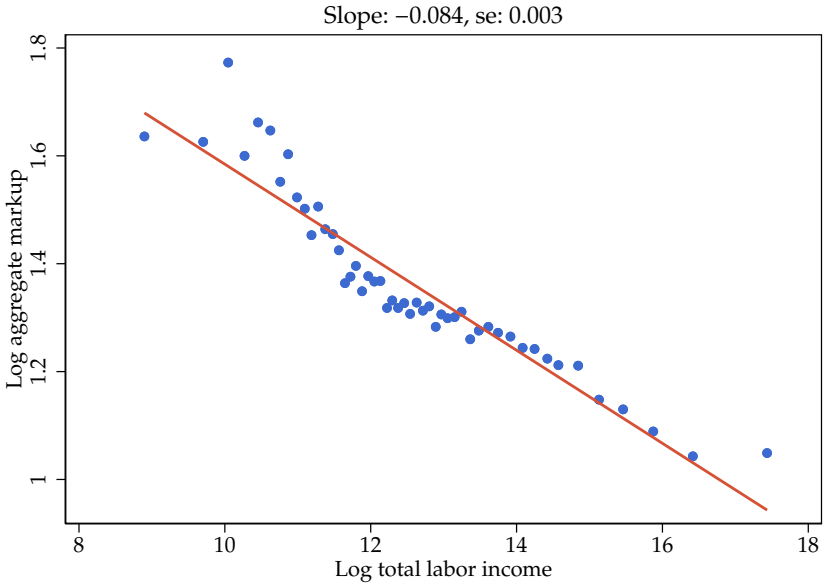
$$\mathcal{M}_c = \left( \sum_{j \in c} s_{jc} \frac{1}{\mu_{jc}} \right)^{-1}, \tag{47}$$

where the sum is taking over the local establishments in county $c$.

Figure 2 shows the relationship between county aggregate markup and county size. The figure displays a clear empirical pattern: bigger counties have a markup significantly lower than their smaller counterparts. Counties like Manhattan or Cook County (Chicago) have a markup 50% lower than small counties like Highland, VA, or Armstrong, TX. Moreover, an increase of 1% in a county's size is associated with a decrease in the county markup of 0.084%. Table 3 shows that empirical findings are not particular to 2017 or defining a city as a county. The negative pattern between county markup and county size is also present when considering other years and defining a city in the data as a Commuting Zone.

Figure 2 sheds light on the mechanisms that shape the distribution of markups across cities. On the one hand, the empirical regularities shown in Section 4.3 show that bigger locations attract more productive producers. However, Figure 2 shows that markups in such locations are significantly lower than in small locations. Through the lens of the theory, the competition force dominates the selection force. Even though bigger cities attract more productive local producers, competition

Figure 2: County aggregate markup and county size




**Notes:** Figure 2 shows a bin-scatter of county log aggregate markup and log total labor income. County size is defined as total labor income. The bin-scatter considers 50 equally sized county bins according to their total labor income. County aggregate markup is a sales-weighted harmonic mean of the local establishment's (47).

in those locations is high enough to restrain the market power of local producers. Furthermore, guided by the results in Proposition 3, this finding suggests that the dispersion of local producers' productivity is lower relative to the local characteristics of cities.

The negative relationship between county markup and county size also sheds light on the spatial misallocation of local producers. Proposition 4 highlights that if markups were higher in bigger cities, the misallocation of local producers would be exacerbated. Nonetheless, Figure 2 sends a reassuring message that markups in larger cities are lower than in smaller cities. The results suggest that the competition force governing local producers' location decisions prevents establishments from over-locating in bigger cities. Competition in bigger cities is intense enough to prevent local producers from charging higher markups and induces marginal producers to locate in smaller cities.

Although the primary goal of the current section is to analyze markups for all local industries, I turn now to a sector-specific analysis of markups across cities. Although I focus on the Retail and Manufacturing, Appendix G.1 shows results for the other sectors.

Figure 3 shows markups across cities for local Retail and Manufacturing. The patterns for Retail resemble the ones from all local industries. Local Retail producers in big cities charge a markup 60% lower than local Retail producers in the smallest cities. This finding is unsurprising as local Retail accounts for one-quarter of local local industries' employment. Hence, it is reasonable to think that local Retail producers are one of the drivers of the dynamics displayed in Figure 2.

Markups for local Manufacturing producers are higher in larger cities. Contrary to the results for all local industries, local Manufacturers in the bigger counties charge a markup two times higher than producers in the smallest cities. Furthermore, the local Manufacturing markup distribution

Table 3: Average City Markup elasticity with respect to City Size

| | Dep. var.: Log aggregate markup | | | |
|---|---|---|---|---|
| | 2002 | 2007 | 2012 | 2017 |
| | (1) | (2) | (3) | (4) |
| | Panel A: Counties | | | |
| Log total labor income | -0.0987*** | -0.0897*** | -0.0931*** | -0.0834*** |
| | (0.0032) | (0.0034) | (0.0033) | (0.0031) |
| Observations | 3100 | 3100 | 3100 | 3100 |
| R-squared | 0.368 | 0.328 | 0.363 | 0.323 |
| | Panel B: Commuting Zones | | | |
| Log total labor income | -0.0671*** | -0.0662*** | -0.0651*** | -0.0609*** |
| | (0.0029) | (0.0026) | (0.0030) | (0.0029) |
| Observations | 750 | 750 | 750 | 750 |
| R-squared | 0.598 | 0.61 | 0.526 | 0.551 |

**Notes:** Table 3 displays the average elasticity of county aggregate markup and city size. City aggregate markups is defined as (47) and city size is defined as total labor income. Panel A shows the mean elasticity defining cities as counties. Panel B shows the mean elasticity defining cities as Commuting Zones. Robust standard errors in parenthesis. *10% level, **5% level, ***1% level.

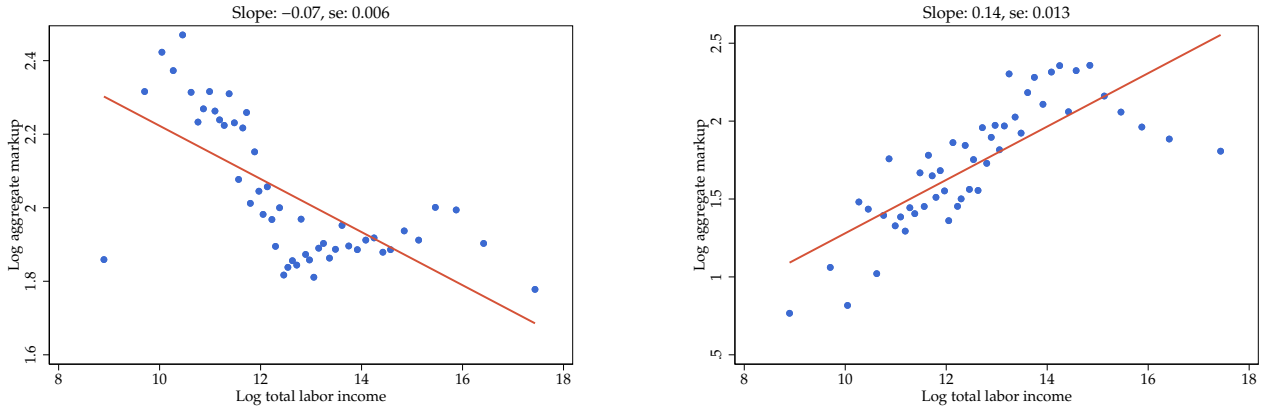
across cities unveils two additional findings. First, the forces that govern competition and selection of local producers seem to vary across sectors. Second, the markup estimation procedure outlined in Section 4.4.2 does not mechanically deliver lower markups in big cities.

The different empirical patterns for local Retail and local Manufacturing inform the spatial misallocation across sectors. Because local manufacturing markups in larger markets are higher, the spatial misallocation in Manufacturing may be more considerable than the misallocation in Retail. The selection force in local Manufacturing is significantly stronger than in local Retail. Intuitively, the productivity differences across local Manufacturing plants are much more significant than those across local Retail producers.

**Robustness.** I use the Manufacturing sector to perform robustness exercises that provide additional empirical support for the findings of this section. In contrast to other censuses, the Census of Manufactures has detailed data on production inputs. In particular, I observe materials, energy, and capital expenditure measures. The detailed data allows me to investigate the variation in markups across cities with two variations of the baseline estimation. Appendix G.2 shows the results of these alternative exercises.

First, one potential threat to the markup estimating equation (45) is that labor might not be fully flexible. I tackle this concern by estimating this equation using materials and energy as flexible inputs. Second, I relax the Cobb-Douglas assumption by considering a general production function. Under this approach, output elasticities are no longer constant and can be a function of

Figure 3: County aggregate markup and county size for Local Retail and Local Manufacturing




(a) Retail (b) Manufacturing

**Notes:** Figure 3(a) shows a bin-scatter of county log aggregate markup for Local Retail and log total labor income. Figure 3(b) shows a bin-scatter of county log aggregate markup for Local Manufacturing and log total labor income. County aggregate markup for Retail is a sales-weighted harmonic mean of the Retail local establishment's (47). County aggregate markup for Manufacturing is a sales-weighted harmonic mean of the Manufacturing local establishment's (47). In both figures, county size is defined as total labor income. Both bin-scatters consider 50 equally sized county bins according to their total labor income.

the production inputs. In particular, I approximate the output elasticity by a flexible polynomial in labor, materials, energy, and capital as in Gandhi, Navarro, and Rivers (2020). Under these two alternative procedures, the estimated markups highly correlate with the baseline estimates. I also obtain the same cross-sectional city variation as in Figure 3(b).

This section outlined the empirical approach to estimating markups using U.S. micro-data and showed the resulting markup estimates. There is significant cross-section heterogeneity in markups across local producers. Moreover, there is also a significant heterogeneity in the markups across cities, with bigger cities having lower markups than smaller ones. The results support the idea that local competition and local producers' productivity vary tremendously across space and, in turn, offer empirical support for the economic forces proposed in the theoretical framework. We now turn to the quantitative investigation that measures the welfare effects of place-based policies.

# 5 Quantitative Analysis

In this section, I estimate the model and use it to quantify the general equilibrium effects of place-based policies. Similarly to the empirical analysis section, I define a city in the model as a county. Focusing on the continental U.S., the quantitative exercises consider 3,080 counties. Even though the different forces highlighted in the model may act differently across sectors, as shown in 3(a), I estimate the parameters for establishments in all local industries. Similarly, the counterfactual exercises abstract from sector heterogeneity across local industries.

Table 4: First Group: External Calibration

| Parameter | Description | Source | Value |
|---|---|---|---|
| $\alpha$ | Housing expenditure share | Davis and Ortalo-Magne (2011) | 0.24 |
| $\phi$ | Housing supply elasticity | Saiz (2010) | 1.75 |
| $\theta$ | Dispersion location preferences | Fajgelbaum et al. (2018) | 1.73 |

## 5.1 Model Estimation

The model has 12 parameters to be estimated, which I divide into three groups. Three parameters in the first group are externally calibrated using standard values from the literature. The second group comprises five parameters estimated using a Generalized Method of Moments (GMM) estimator. The model delivers estimating equations for each of the parameters in the group. Finally, a Simulated Method of Moments (SMM) routine estimates four parameters in the third group. I target the establishment's average employment across counties displayed in Figure 1(b) and the economy-wide aggregate markup.

City exogenous characteristics, traded good productivity and amenities, are recovered non parametrically by exactly matching employment and average wages per county.[55]

**Externally Calibrated Parameters (3 parameters).** This group has three parameters: the housing expenditure share $\alpha$, buildings supply elasticity $\phi$, and the idiosyncratic location preference tastes dispersion, $\theta$.

Table 4 summarizes the values for the three parameters. The housing expenditure share takes the value reported by Davis and Ortalo-Magne (2011), $\alpha = 0.24$. The housing supply elasticity is set to $\phi = 1.75$, the unweighted median elasticity of Saiz (2010). Finally, I set the dispersion of the location idiosyncratic preference tastes to $\theta = 1.73$, which is the baseline value estimated by Fajgelbaum et al. (2018) for the U.S.

**GMM Estimated Parameters (5 parameters).** There are five parameters in this group: the local goods expenditure share $\eta$, the Kimball demand parameters $\overline{\sigma}$, $\varepsilon$, and the local and traded good producer output elasticities, $\beta$ and $\gamma$. Table 5 summarizes the results.

The local goods expenditure share, $\eta$, is estimated using the local goods sales share reported in Table 1. Given the housing expenditure share value, $\alpha = 0.24$, an aggregate sales share of 52% implies a value of $\eta = 0.39$. Letting $sales_j$ denote the sales of establishment $j$, $\hat{\eta}$ is formally estimated by:

$$\hat{\eta} = (1-\alpha) \times \underset{\eta}{\operatorname{argmin}} \left\| \frac{\sum_{j \in \text{Local}} sales_j}{\sum_j sales_j} - \eta \right\| \tag{48}$$

I estimate the Kimball demand parameters in two steps. First, I use the relationship between the markups estimated and the establishment sales shares to estimate the ratio $\varepsilon/\overline{\sigma}$. Appendix E.1

[55] For the counterfactual exercises, I use the non-parametric estimates to fit a joint log normal distribution for $a$ and $b$.

Table 5: Second Group: GMM

| Parameter | Description | Moment | Estimate |
|---|---|---|---|
| $\eta$ | Local goods expenditure share | Local establishments sales share | 0.39 |
| $\varepsilon$ | Demand super-elasticity | Markups and sales share | 1.38 |
| $\overline{\sigma}$ | Demand elasticity | Markups and implied relative quantities | 2.26 |
| $\beta$ | Labor output elasticity (local) | Local establishments labor FOC | 0.22 |
| $\gamma$ | Labor output elasticity (traded) | Traded establishments labor FOC | 0.29 |

shows that the Klenow and Willis (2016) Kimball specification implies the following relationship between establishment $j$ markup in county $c$, $\mu_{jc}$, and the establishments sales share, $s_{jc}$:

$$\frac{1}{\mu_{jc}} + \log\left(1 - \frac{1}{\mu_{jc}}\right) = \frac{\overline{\sigma} - 1}{\overline{\sigma}} - \log\overline{\sigma} + \frac{\varepsilon}{\overline{\sigma}}\log\frac{\overline{\sigma}}{\overline{\sigma} - 1} - \frac{\epsilon}{\overline{\sigma}}\log\frac{P_c}{D_c} + \frac{\epsilon}{\overline{\sigma}}\log s_{jc}, \tag{49}$$

Using the estimated markups, $\hat{\mu}_{jc}$, I estimate the following equation via OLS:

$$\frac{1}{\hat{\mu}_{jc}} + \log\left(1 - \frac{1}{\hat{\mu}_{jc}}\right) = \varpi + \varpi_c + \frac{\epsilon}{\overline{\sigma}}\log s_{jc} + \iota_{jc}, \tag{50}$$

where $\varpi$ is a constant absorbing the constant terms in (49), $\varpi_c$ is a county fixed-effect absorbing the term $(\epsilon/\overline{\sigma})\log(P_c/D_c)$, and $\iota_{jc}$ is an approximation error coming from the estimation of the markups. The regression coefficient of $\log s_{jc}$ is an estimate of the ratio $\varepsilon/\overline{\sigma}$.

Equipped with an estimate of the ratio $\varepsilon/\overline{\sigma}$, I develop an iterative GMM procedure that estimates $\overline{\sigma}$. Using a similar logic to (41), the Kimball demand implies the following system of equations for relative quantities and price indices as functions of revenue market shares:

$$\begin{aligned}
\frac{y_{jc}}{Y_c} &= \left[-\overline{\sigma}\Omega\left(-\left(s_{jc}\frac{P_c}{D_c}\frac{\overline{\sigma}}{\overline{\sigma} - 1}\right)^{\frac{\epsilon}{\overline{\sigma}}}\frac{\exp\left(-\frac{1}{\overline{\sigma}}\right)}{\overline{\sigma}}\right)\right]^{\frac{\overline{\sigma}}{\epsilon}}, \\
\frac{P_c}{D_c} &= \sum_{i \in c}\frac{\overline{\sigma} - 1}{\overline{\sigma}}\exp\left(\frac{1 - \frac{y_{jc}}{Y_c}^{\frac{\varepsilon}{\overline{\sigma}}}}{\varepsilon}\right)\frac{y_{jc}}{Y_c}.
\end{aligned}$$

With the $\varepsilon/\overline{\sigma}$ estimate, the data on sales share, and a given value of $\overline{\sigma}$, the above system gives relative quantities $\frac{y_{jc}}{Y_c}(\overline{\sigma})$. With the implied relative quantities, I compute the implied markups:

$$\breve{\mu}_{jc}(\overline{\sigma}) = \frac{1}{1 - \frac{1}{\overline{\sigma}}\frac{y_{jc}}{Y_c}(\overline{\sigma})^{\frac{\varepsilon}{\overline{\sigma}}}}$$

Then, I estimate $\overline{\sigma}$ such that we minimize the distance between the predicted and the estimated markups:

Table 6: Third Group: SMM

| Moments | | Model | Data |
|---|---|---|---|
| Ratio avg. estab. employment p25/p5 | | 1.62 | 1.31 |
| Ratio avg. estab. employment p50/p5 | | 1.67 | 1.50 |
| Ratio avg. estab. employment p75/p5 | | 1.80 | 1.87 |
| Ratio avg. estab. employment p95/p5 | | 2.21 | 2.23 |
| Aggregate markup | | 2.19 | 3.28 |
| Description | Parameter | Estimate | |
| Min. productivity | $z_L$ | 10.61 | |
| Max. productivity | $z_H$ | 59.44 | |
| Shape parameter | $\delta$ | 4.850 | |
| Entry cost | $c_e$ | 0.041 | |

$$\widehat{\overline{\sigma}} = \underset{\overline{\sigma}}{\operatorname{argmin}} \left\| \check{\mu}_{jc}(\overline{\sigma}) - \hat{\mu}_{jc} \right\|$$

Lastly, we turn to the estimation of the labor output elasticities. For establishments in local industries, I recover an estimate of $\beta$ from the markup estimation equation (45). For establishments in traded industries, I estimate $\gamma$ from (23), which states that $\gamma$ equals the labor cost share of sales. Formally, letting $wagebill_j$ be establishments $j$ wage bill, $\hat{\gamma}$ solves:

$$\hat{\gamma} = \underset{\gamma}{\operatorname{argmin}} \left\| \frac{\sum_{j \in \text{Traded}} wagebill_j}{\sum_{j \in \text{Traded}} sales_j} - \gamma \right\| \tag{51}$$

**SMM Estimated Parameters (four parameters).** The parameters in the last group are the ones governing the local producers' productivity distribution in (29), $z_L$, $z_H$, and $\delta$, in addition to the entry cost $c_e$. Table 6 displays the estimated parameters and the goodness of fit.

I estimate the four parameters via SMM. I target the establishment's average employment reported in Table 1(b). However, to avoid taking a stand on the units in which labor is measured in the model, I target the average establishment employment for counties in the 25th, 50th, 75th, and 95th percentiles relative to the average establishment employment in the 5th percentile. This yields four moments. Additionally, I target the economy-wide aggregate markup for local industries. Following the insights of Yeh, Macaluso, and Hershbein (2022), I define the economy-wide markup as a population-weighted average of the county-level aggregate markups. I compute this object using the markup estimates from Section 4. In total, I am over-identified by having five moments and four parameters.

Formally, let $\Theta = (z_L, z_H, \delta, c_e)$ be the vector of parameters to estimate. I implement the SMM by minimizing the squared percent distance between the model-simulated moments, $M^m(\Theta)$, and their empirical counterparts, $M^d$:

$$\min_{\Theta} \sum_{i=1}^{5} \left( \frac{M_i^m(\Theta) - M_i^d}{0.5\left(M_i^m(\Theta) + M_i^d\right)} \right)^2 .$$

I employ the TikTak algorithm for global optimization of Arnoud, Guvenen, and Kleineberg (2019) to search over the parameter space.[56] In every iteration of the optimization routine, I invert the model by non-parametrically estimating $a_c$ and $b_c$ using (11) and (22). Appendix F discusses the inversion procedure.

Even though all parameters are jointly identified, it is possible to shed light on which moments help to identify each parameter. The counties' average employment relative to the smaller counties identifies the local producers' productivity distribution parameters: $z_L$, $z_H$, and $\delta$. Conversely, the aggregate markup identifies the entry cost.

Table 6 shows the goodness of fit of the SMM estimation. The model is flexible enough to match the average employment of counties in the 25th, 50th, 75th, and 95th percentiles of county size distribution relative to counties in the 5th percentile. Nevertheless, the model falls short when matching the aggregate markup. Solving the decentralized equilibrium involves challenging fixed point algorithms with systems of highly non-linear equations within them. Improving the match between the model aggregate markup and the one estimated in the data is still a work in progress.

## 5.2 Model to the Data: the Decentralized Equilibrium

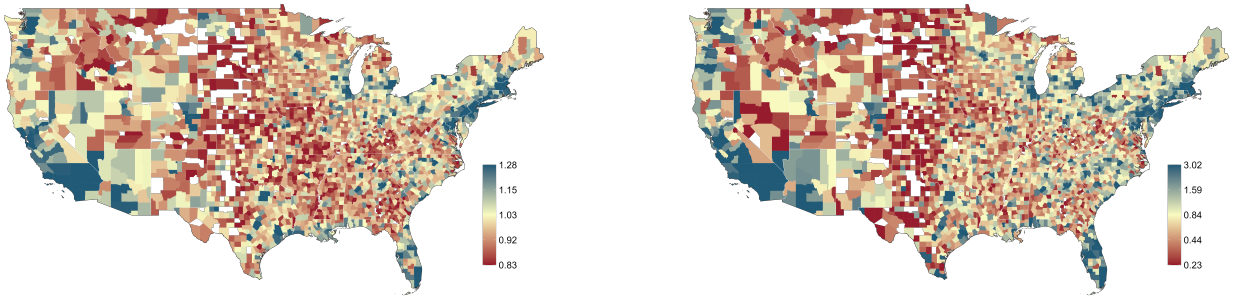
In this section, I solve for the decentralized equilibrium using the estimated parameters and show that the model can quantitatively account for spatial markup differences.

First, Figure 4 displays the results from the model inversion. Local productivity of the traded good is displayed on the left panel, while local amenities are displayed on the right. Overall, both county characteristics are highly correlated. Because the inversion exactly matches wages and population, the model rationalizes high-wage counties as having high traded good productivity. These counties are typically in the upper east coast, southern Florida, the Midwest, and southern California. On the other hand, counties with large populations are rationalized to have higher local amenities. In contrast to local productivity, a significant fraction of the southern counties have high local estimated amenities.

Second, Figure 5 displays the model implied markups. It is worth mentioning that even though the economy-wide aggregate markup is one of the targeted moments, the markup's cross-sectional variation is not constrained by the estimation. Therefore, this figure serves as an over-identifying exercise. Figure 5(a) is the model equivalent of Figure 2. On the one hand, the model can qualitatively replicate the negative relationship between county aggregate markup and county size. However, on the other hand, the model estimated elasticity of aggregate markup with respect to county size is -0.048, which is lower than the one estimated with the data. Indeed, there is a level effect that the model fails to capture, and therefore, it predicts markups somewhat low. Nonetheless, the model can capture the relative markup difference between the smallest and biggest

[56] I use 2000 starting points and a simplex search method for the local optimization.

Figure 4: Local Productivity and Local Amenities




(a) Local Productivity (b) Local Amenities

**Notes:** Figure 4(a) shows the model implied local traded good productivity and Figure 4(b) shows the model implied local amenities. Counties omitted in the analysis are not colored.

counties illustrated in Figure 2: counties in the top decile of total labor income distribution exhibit a markup 50% lower than counties in the bottom decile.

Figure 5(b) shows the model implied markups for all counties in the U.S. Small counties typically located in the south and central parts of the U.S. have markups from 2.17 to 3.17. On the contrary, big counties like Manhattan, Chicago, or Los Angeles display markups that are almost twice as small as those in small locations.

Figure 6 shows that the model can qualitatively account for different empirical regularities across cities. The blue solid line across panels illustrates different economic outcomes in decentralized equilibrium (Laissez-faire case). First, in line with the findings of Combes et al. (2012), the model predicts that bigger cities are more productive. The productivity advantage of bigger cities comes through two channels: it attracts more productive firms and displays higher local TFP. Recall that local TFP accounts for the productivity of local producers, but also it increases with the number of firms in a location. Indeed, TFP in bigger counties doubles TFP in smaller counties. Second, the bottom-left panel displays the local varieties price index, $\mathbb{P}(c)$. Consistent with the findings of Handbury and Weinstein (2014), bigger cities have a lower price index. The model also accounts for the fact that bigger counties have higher housing rents.

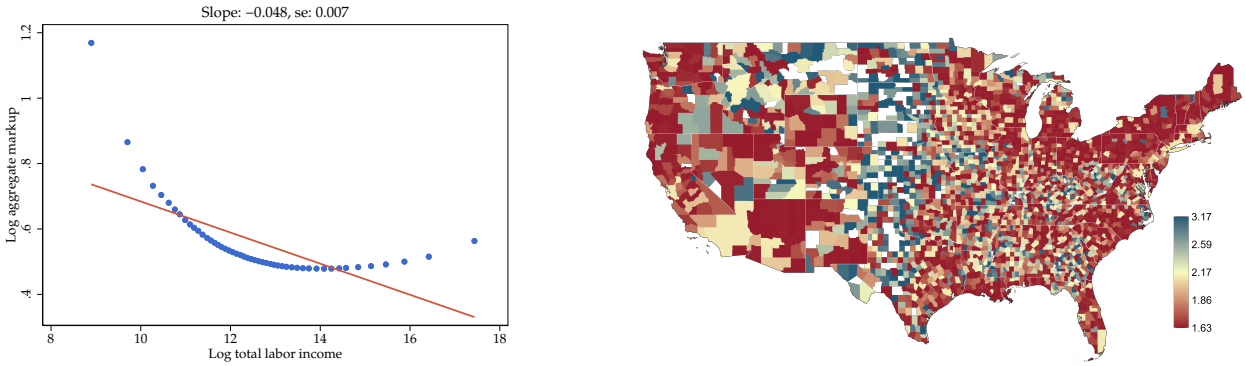
Figure 6 also shows the cross-sectional differences in local competition across counties. The top-right panel documents a significant heterogeneity in the competition index across counties in the estimated model. As illustrated by the central panels at the bottom of the figure, the tougher competition in these locations is partially explained by intense competition in the inputs markets. Furthermore, firm prices in such locations are also lower than in smaller counties, magnifying the differences in the local competition index.

## 5.3 Place-based Policy Counterfactual

This final section studies a policy counterfactual. Formally, I investigate the aggregate effects of implementing the optimal policy of Proposition 5.

The location-specific subsidy (34) implements the optimal policy and achieves the first-best alloca-

Figure 5: Markups in the Decentralized Equilibrium




(a) County agg. markup and county size (b) Markups across counties

**Notes:** Figure 5(a) shows the model equivalent of Figure 2. Figure 5(b) shows markups across counties in the decentralized equilibrium. Counties omitted in the analysis are not colored.

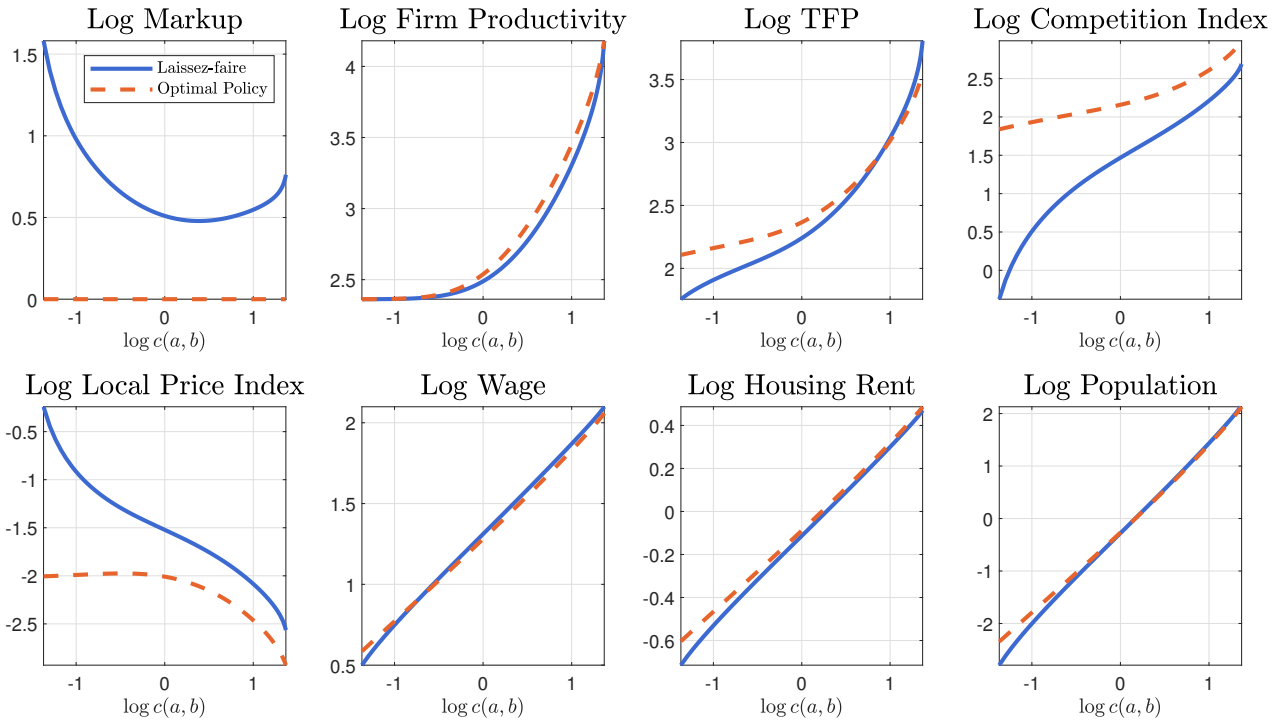
tion. Recall that this transfer corrects three margins of inefficiency in the decentralized equilibrium: removes output price distortions, corrects the inefficient location of local producers, and generates an efficient aggregate entry rate. A non-distortionary tax on workers finances this subsidy. Moreover, I use the equivalent formulation of the baseline model in which profits from local developers are rebated back to workers in a non-distortionary flat earnings subsidy.

Figure 6 displays the cross-sectional patterns of the equilibrium under the laissez-faire and the optimal policy. Consistent with the results from Proposition 5, the optimal policy removes markups in all locations. Furthermore, the policy makes marginally productive producers relocate to smaller locations. More than 90% of the counties experience a productivity boost due to this policy. Nonetheless, this comes at the expense of productivity losses in larger locations. By removing markups, the policy also makes the price index fall everywhere. Nevertheless, the price index in smaller counties experience a more prominent decrease because of two channels. First, these locations initially had higher markups and, therefore, experience more considerable reductions in prices. Second, as these locations experience an influx of producers, the increase in local varieties also causes the price index to fall relatively more than in larger locations.

As a result of the policy, smaller cities expand. The bottom half of Figure 6 shows that smaller cities experience an increase in wages, housing rents, and population. The spatial reallocation of firms increases labor demand in smaller cities, which creates an upper pressure on wages and causes a relocation of workers to such locations. In larger counties, this spatial reconfiguration slightly decreases wages but has milder effects on population. Interestingly, housing rents increase in all locations. The reason is that reducing markups induces firms to increase production and augment their input demand. As firms demand more commercial structures, housing rents rise.

To highlight the spatial effects of the optimal policy, Figure 7 maps changes at the local level for different equilibrium outcomes. First, Figure 7(a) shows the change in productivity of local producers. There are two crucial messages this graph conveys. First, the local productivity level in the biggest and smallest counties remains unchanged. Local producers in Manhattan or rural

Figure 6: Model's solution in the Decentralized Equilibrium and the Optimal Policy



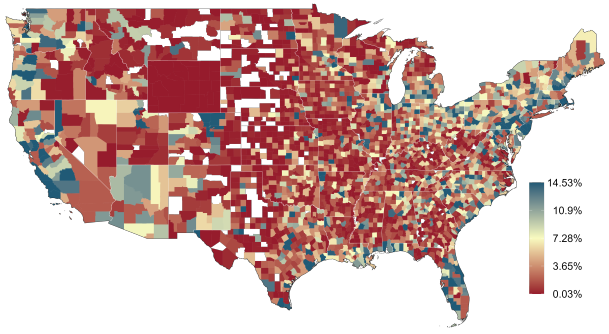

**Notes:** Figure 6 shows model solution in the decentralized equilibrium and under the optimal policy for all U.S. local industries. The x-axis represents the city's estimated appeal $c(a, b)$.

small counties do not change their location decisions due to the policy. The reason is that positive assortative matching still holds in the optimal policy. Therefore, the very best producers are still located in the bigger cities, and the very least productive in the smallest. Second, counties that experience the most significant productivity increases are those located next to the most extensive locations. Indeed, the productivity of local producers in adjacent counties to large urban areas like Miami, Chicago, and Los Angeles increases by almost 15%. This reflects the "top-down" effect of the optimal policy: marginal producers relocate from big cities to marginally smaller ones. This reallocation effect also occurs once we move down across smaller counties.

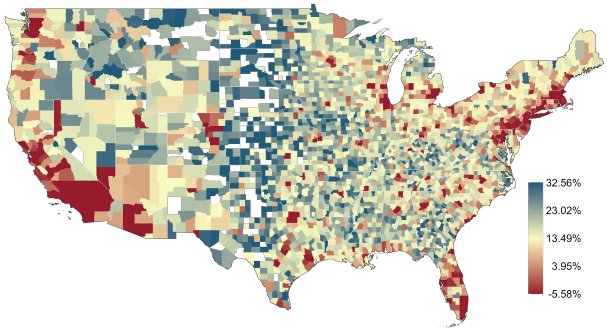
Figure 7(b) shows the change in local TFP. In addition to accounting for the productivity level of local producers, total TFP also accounts for the total number of firms. Interestingly, as the policy also changes the number of producers in each city, changes in local productivity are magnified through the number of local producers, which leads to more significant changes in total TFP. On the one hand, smaller counties, primarily located in the inner part of the country, experience a significant increase in TFP. For instance, Floyd, TX, reaches a productivity increase of 30%. On the other hand, highly populated counties like Los Angeles suffer a mild decrease of around 5%. Changes in the number of producers primarily drive changes in TFP for the biggest and smallest counties. Interestingly, Figure 7(b) shows the pattern that rural counties seem to be the ones that benefit the most from the policy. Along these lines, the policy speaks to the discussion of Urban vs Rural development, suggesting that the policies that aim to boost commercial activity in under-developed rural areas may be beneficial.

Figure 7(c) displays the spatial heterogeneity in local prices. Counties in the coastal parts of the

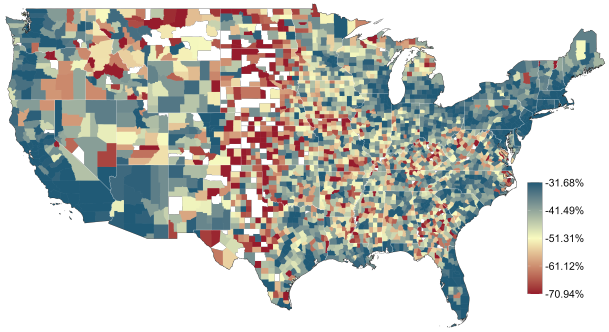
Figure 7: Changes in firm productivity, local TFP, competition index and ideal price index.



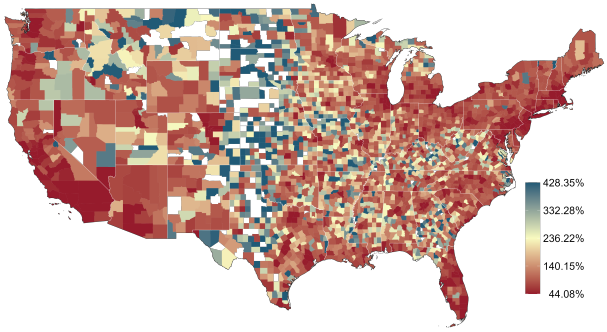

(a) Firm Productivity, $z(c)$




(b) Local TFP, $\mathcal{Z}(c)$




(c) Ideal Price Index, $\mathbb{P}(c)$




(d) Competition Index, $\mathbb{C}(c)$.

**Notes:** Figures show the percentual changes in local equilibrium objects in the optimal policy relative to the Laissez-faire scenario. Figure 7(a) shows the percentual change in local firm productivity. Figure 7(b) displays the percentual change in local TFP. Figure 7(d) illustrates the percentual change in the competition index, and Figure 7(c) describes the percentual change in the ideal price index.

country and around the Great Lakes region experience a reduction in the local prices of around 30%. Strikingly, southern and central counties witness a reduction in the price index close to 70%. There are two reasons behind this significant disparity in price reduction. First, the small counties are experiencing a more considerable reduction in markups. In such locations, markups go from 3.1 to 1. Second, these counties are the ones experiencing a large influx of new local producers, which further decreases the local price index.

The last panel, Figure 7(d), displays the change in the competition index $\mathbb{C}(c)$. Recall that the index $\mathbb{C}(c)$ has two components. On the one hand, it reflects the prices of local producers. As seen from Figure 7(c), local prices fall everywhere because firms no longer charge a markup over marginal cost. Therefore, local competition increases everywhere as local producers charge lower prices. On the other hand, the competition index also captures the price of the local inputs: labor and structures. Because the policy incentivizes firms to increase production, all firms demand more of these inputs, causing wages and land rents to increase. The rise in local input prices further increases competition in all counties. Nonetheless, similarly to the pattern in Figure 7(c), small counties experience a more considerable increase in competition as they experience higher price reductions.

The results from the counterfactual exercise illustrate the benefits of a policy that reallocates producers from big to small cities. Qualitatively, these results are similar to the ones in Bilal (2023). Nevertheless, the rationale for such types of policy in Bilal (2023) is different from the ones considered in this study. While the rationale in Bilal (2023) comes from labor market frictions, I offer theoretical and empirical support for these policies based on the premise that output market power causes local producers to locate inefficiently. Both papers contrast with the findings Gaubert (2018), who finds that incentivizing producers to locate in smaller locations is detrimental because of agglomeration externalities. A potentially interesting future research avenue would be explicitly combining to explicitly combine all these mechanisms and asses their relative importance.

Finally, the optimal policy yields an aggregate welfare gain of 2.36%. This magnitude is within the range of results of both Bilal (2023) and Gaubert (2018). However, the gain in welfare is lower than in Edmond, Midrigan, and Xu (2023). Of course, the framework they consider differs significantly from the one in this paper, with the spatial component being the key difference.

# 6 Conclusion

This paper has developed a new theory of endogenous competition across cities. The theory sheds light on the mechanisms that govern the ability of local producers to exert output market power. Differences in markups across space arise due to differentials in local competition and the productivity of local firms. Pricing complementarities are the central driver of the location choice of heterogeneous producers. As a result, more productive firms over-value locating in bigger cities, and spatial misallocation arises. This view emphasizes that relocating production from bigger to smaller cities increases aggregate welfare.

The paper also provides empirical evidence on markup heterogeneity across the U.S. The structure of my model allows me to estimate markups for all the establishments that operate in local markets. Producers in larger cities have significantly lower markups than producers in smaller cities. This empirical regularity is informative of the degree of firm spatial misallocation.

Finally, I use the model to quantify the welfare gains of place-based policies. Policies that eliminate markups yield sizable welfare gains by eliminating price distortion and relocating firms from big to small cities. The view that output market power creates firm misallocation across cities helps to reconcile the intuition of place-based policies.

The methodology proposed in this paper can readily be used to study the determinants of local competition across different sectors. I empirically illustrate that different sectors have different patterns for markups across cities. These patterns suggest that the magnitude of the economic forces that determine firm location and local competition might differ across sectors. This has implications for the degree of spatial misallocation across sectors. The degree of firm misallocation across cities would be worse in sectors where markups are higher in bigger cities. Studying general equilibrium counterfactuals for different sectors is left for future research. A quantitative assessment of the general equilibrium effects of place-based policies for different sectors could be used to inform industrial policy.

# References


Ackerberg, Daniel et al. (2007). "Econometric Tools for Analyzing Market Outcomes". In: *Handbook of Econometrics*. Ed. by J.J. Heckman and E.E. Leamer. 1st ed. Vol. 6A. Elsevier. Chap. 63.

Ackerberg, Daniel A., Kevin Caves, and Garth Frazer (2015). "Identification Properties of Recent Product Function Estimators". In: *Econometrica* 83.6, pp. 2411–2451.

Aghion, Philippe et al. (Feb. 2023). "A Theory of Falling Growth and Rising Rents". In: *The Review of Economic Studies*, rdad016.

Akcigit, Ufuk and Sina T. Ates (2023). "What Happened to US Business Dynamism?" In: *Journal of Political Economy* 131.8, pp. 2059–2124.

Amiti, Mary, Oleg Itskhoki, and Jozef Konings (Feb. 2019). "International Shocks, Variable Markups, and Domestic Prices". In: *The Review of Economic Studies* 86.6, pp. 2356–2402.

Anderson, Eric, Sergio Rebelo, and Arlene Wong (Mar. 2018). *Markups Across Space and Time*. Working Paper 24434. National Bureau of Economic Research.

Arellano, Manuel and Olympia Bover (1995). "Another Look at the Instrumental Variable Estimation of Error-Components Models". In: *Journal of Econometrics* 68.1, pp. 29–51.

Arnoud, Antoine, Fatih Guvenen, and Tatjana Kleineberg (Oct. 2019). *Benchmarking Global Optimizers*. Working Paper 26340. National Bureau of Economic Research.

Atalay, Enghin (2014). "MATERIALS PRICES AND PRODUCTIVITY". In: *Journal of the European Economic Association* 12.3, pp. 575–611. (Visited on 10/25/2023).

Atkeson, Andrew and Ariel Burstein (Dec. 2008). "Pricing-to-Market, Trade Costs, and International Relative Prices". In: *American Economic Review* 98.5, pp. 1998–2031.

Autor, David H. and David Dorn (Aug. 2013). "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market". In: *American Economic Review* 103.5, pp. 1553–97.

Baqaee, David Rezza, Emmanuel Farhi, and Kunal Sangani (June 2023). "The Darwinian Returns to Scale". In: *The Review of Economic Studies*, rdad061.

Berger, David, Kyle Herkenhoff, and Simon Mongey (Apr. 2022). "Labor Market Power". In: *American Economic Review* 112.4, pp. 1147–93.

Berry, Steven, Martin Gaynor, and Fiona Scott Morton (Aug. 2019). "Do Increasing Markups Matter? Lessons from Empirical Industrial Organization". In: *Journal of Economic Perspectives* 33.3, pp. 44–68.

Bilal, Adrien (Mar. 2023). "The Geography of Unemployment*". In: *The Quarterly Journal of Economics* 138.3, pp. 1507–1576.

Blundell, Richard and Stephen Bond (1998). "Initial Conditions and Moment Restrictions in Dynamic Panel Data Models". In: *Journal of Econometrics* 87.1, pp. 115–143.

– (2000). "GMM Estimation with Persistent Panel Data: An Application to Production Functions". In: *Econometric Reviews* 19.3, pp. 321–340.

Bond, Steve et al. (2021). "Some unpleasant markup arithmetic: Production function elasticities and their estimation from production data". In: *Journal of Monetary Economics* 121, pp. 1–14.

Chen, Xiaohong (2007). "Chapter 76 Large Sample Sieve Estimation of Semi-Nonparametric Models". In: ed. by James J. Heckman and Edward E. Leamer. Vol. 6. Handbook of Econometrics. Elsevier, pp. 5549–5632.

Combes, Pierre-Philippe et al. (2012). "The Productivity Advantages of Large Cities: Distinguishing Agglomeration From Firm Selection". In: *Econometrica* 80.6, pp. 2543–2594.

Costinot, Arnaud and Jonathan Vogel (2010). "Matching and Inequality in the World Economy". In: *Journal of Political Economy* 118.4, pp. 747–786.

Davis, Morris A. and Francois Ortalo-Magne (Apr. 2011). "Household Expenditures, Wages, Rents". In: *Review of Economic Dynamics* 14.2, pp. 248–261.

De Loecker, Jan (2011). "Product Differentiation, Multiproduct Firms, and Estimating the Impact of Trade Liberalization on Productivity". In: *Econometrica* 79.5, pp. 1407–1451.

De Loecker, Jan, Jan Eeckhout, and Gabriel Unger (Jan. 2020). "The Rise of Market Power and the Macroeconomic Implications*". In: *The Quarterly Journal of Economics*.

De Loecker, Jan and Chad Syverson (2021). "An Industrial Organization Perspective on Productivity". In: *Working Paper*.

De Loecker, Jan and Frederic Warzynski (May 2012). "Markups and Firm-Level Export Status". In: *American Economic Review* 102.6, pp. 2437–71.

Delgado, Mercedes, Michael E. Porter, and Scott Stern (June 2015). "Defining clusters of related industries". In: *Journal of Economic Geography* 16.1, pp. 1–38.

Edmond, Chris, Virgiliu Midrigan, and Daniel Yi Xu (2023). "How Costly Are Markups?" In: *Journal of Political Economy* 131.7, pp. 1619–1675.

Fajgelbaum, Pablo D et al. (Sept. 2018). "State Taxes and Spatial Misallocation". In: *The Review of Economic Studies* 86.1, pp. 333–376.

Foster, Lucia, Cheryl Grim, and John Haltiwanger (2016). "Reallocation in the Great Recession: Cleansing or Not?" In: *Journal of Labor Economics* 34.S1, S293–S331.

Foster, Lucia S, John C Haltiwanger, and Chad A Syverson (Mar. 2008). "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" In: *American Economic Review* 98.1, pp. 394–425.

Galichon, Alfred (2016). *Optimal transport methods in economics*. Princeton: Princeton University Press.

Gandhi, Amit, Salvador Navarro, and David Rivers (2020). "On the Identification of Gross Output Production Functions". In: *Journal of Political Economy* 128.8, pp. 2973–3016.

Gaubert, Cecile (2018). "Firm Sorting and Agglomeration". In: *American Economic Review* 108.11, pp. 3117–53.

Hall, Robert E (1988). "The Relation between Price and Marginal Cost in U.S. Industry". In: *Journal of Political Economy* 96.5, pp. 921–47.

Handbury, Jessie and David E. Weinstein (Sept. 2014). "Goods Prices and Availability in Cities". In: *The Review of Economic Studies* 82.1, pp. 258–296.

Hottman, Colin (2021). *Retail Markups, Misallocation, and Store Variety across U.S. Cities*. Tech. rep.

Hsieh, Chang-Tai and Peter J. Klenow (Nov. 2009). "Misallocation and Manufacturing TFP in China and India*". In: *The Quarterly Journal of Economics* 124.4, pp. 1403–1448.

Hsieh, Chang-Tai and Esteban Rossi-Hansberg (2023). "The Industrial Revolution in Services". In: *Journal of Political Economy Macroeconomics* 1.1, pp. 3–42.

Kimball, Miles S. (1995). "The Quantitative Analytics of the Basic Neomonetarist Model". In: *Journal of Money, Credit and Banking* 27.4, pp. 1241–1277.

Kleinman, Benny (2023). "Wage Inequality and the Spatial Expansion of Firms". In.

Klenow, Peter J. and Jonathan L. Willis (2016). "Real Rigidities and Nominal Price Changes". In: *Economica* 83.331, pp. 443–472.

Levinsohn, James and Amil Petrin (Apr. 2003). "Estimating Production Functions Using Inputs to Control for Unobservables". In: *Review of Economic Studies* 70, pp. 317–341.

Mankiw, N. Gregory and Michael D. Whinston (1986). "Free Entry and Social Inefficiency". In: *The RAND Journal of Economics* 17.1, pp. 48–58. (Visited on 10/21/2023).

Marshall, Alfred (1890). *Principles of Economics*. London: Macmillan and Co.

Matsuyama, Kiminori and Phillip Ushchev (2017). "Beyond CES: Three Alternative Classes of Flexible Homothetic Demand Systems". In.

– (2021). "When Does Procompetitive Entry Imply Excessive Entry". In.

– (2022). "Selection and Sorting of Heterogeneous Firms Through Competitive Pressures". In.

Melitz, Marc J. (2003). "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity". In: *Econometrica* 71.6, pp. 1695–1725. (Visited on 10/21/2023).

– (2018). "Competitive effects of trade: theory and measurement". In: *Review of World Economics* 154.1.

Melitz, Marc J. and Gianmarco I. P. Ottaviano (Jan. 2008). "Market Size, Trade, and Productivity". In: *The Review of Economic Studies* 75.1, pp. 295–316.

Oberfield, Ezra et al. (2023). "Plants in Space". In: *Journal of Political Economy* 0.ja, null.

Olley, G. Steven and Ariel Pakes (1996). "The Dynamics of Productivity in the Telecommunications Industry". In: *Econometrica* 64.6, pp. 1263–1297.

Peters, Michael (2020). "Heterogeneous Markups, Growth, and Endogenous Misallocation". In: *Econometrica* 88.5, pp. 2037–2073.

Redding, Stephen J. and Esteban Rossi-Hansberg (2017). "Quantitative Spatial Economics". In: *Annual Review of Economics* 9.1, pp. 21–58.

Restuccia, Diego and Richard Rogerson (2008). "Policy distortions and aggregate productivity with heterogeneous establishments". In: *Review of Economic Dynamics* 11.4, pp. 707–720.

Rossi-Hansberg, Esteban, Pierre-Daniel Sarte, and Nicholas Trachter (2020). "Diverging Trends in National and Local Concentration". In.

Saiz, Albert (Aug. 2010). "The Geographic Determinants of Housing Supply*". In: *The Quarterly Journal of Economics* 125.3, pp. 1253–1296.

Yeh, Chen, Claudia Macaluso, and Brad Hershbein (July 2022). "Monopsony in the US Labor Market". In: *American Economic Review* 112.7, pp. 2099–2138.

# A Derivations

## A.1 Local Goods Demand

For any given $Y(c)$ consumer minimize total expenditure on local goods, subject to the utility constraint (2):

$$\mathcal{L}^W = \int_z p(z,c)y(z,c)dG_c(z) + \lambda(c)\left[1 - \int_z \Upsilon\left(\frac{y(z,c)}{Y(c)}\right)dG_c(z)\right],$$

where $\lambda^W(c)$ is a Lagrange multiplier. The first order condition with respect to the consumption of a single variety $y(z,c)$ is:

$$p(z,c) = \frac{\lambda^W(c)}{Y(c)}\Upsilon'\left(\frac{y(z,c)}{Y(c)}\right)$$

Defining the competition price index as:

$$\mathbb{D}(c) \equiv \frac{\lambda(c)}{Y(c)},$$

we can write the inverse demand for a single variety $y(z,c)$ as:

$$\frac{p(z,c)}{\mathbb{D}(c)} = \Upsilon'\left(\frac{y(z,c)}{Y(c)}\right)$$

Similarly, defining $\varphi(\cdot) \equiv (\Upsilon')^{-1}(\cdot)$, the demand function for a single variety $\omega$ is:

$$\frac{y(z,c)}{Y(c)} = \varphi\left(\frac{p(z,c)}{\mathbb{D}(c)}\right).$$

With these expressions, the competition price index $\mathbb{D}(c)$ is given by:

$$\int_z \Upsilon\left(\varphi\left(\frac{p(z,c)}{\mathbb{D}(c)}\right)\right)dG_c(z) = 1$$

Finally, we can express the ratio of $\mathbb{D}(c)$ and $\mathbb{P}(c)$ as a function or relative prices as

$$\frac{\mathbb{P}(c)}{\mathbb{D}(c)} = \int_z \frac{p(z,c)}{\mathbb{D}(c)}\varphi\left(\frac{p(z,c)}{\mathbb{D}(c)}\right)dG_c(z) \tag{A1}$$

## A.2 Input Demands

Input demands follow from the firms cost minimization problem. Taking input prices as given, firms minimize total input expenditure subject to a certain level of production. The Lagrangian associated to this problem is:

$$\mathcal{L}^F = W(c)l(z,c) + R(c)s(z,c) + \lambda(z,c)\left[y(z,c) - zl(z,c)^{\beta}s(c,z)^{1-\beta}\right],$$

where $\lambda(z,c)$ is a Lagrange multiplier that equals the marginal cost of production. Taking first-order conditions and recognizing that $\mu(z,c) \equiv p(z,c)/\lambda(z,c)$ we obtain:

$$l(z,c)W(c) = \beta\frac{p(z,c)y(z,c)}{\mu(z,c)}, \qquad s(z,c)R(c) = (1-\beta)\frac{p(z,c)y(z,c)}{\mu(z,c)}$$

Substituting in the optimal relative price, $\psi(\mathbb{C}(c)/z)$, and the optimal relative quantities $\varphi(\psi(\mathbb{C}(c)/z))$ gives (19).

# B Proofs

## B.1 Proposition 1

The proof of Proposition 1 is structured in three steps. First, we show that there is positive assortative matching conditional on the location's competition index $\mathbb{C}(c)$. Second, we characterized general equilibrium objects when firms sort based on competition. Third, we construct a location's single index which is a sufficient statistic for the firm's location decisions. Then we show that there is positive assortative matching when the location's competition index is determined in general equilibrium.

**Step 1: sorting conditional on competition index.** Note that the optimal relative price and hence the optimal markup depend directly on the location $c$ only through the aggregate object $\mathbb{C}(c)$. Hence, following the insights of Bilal (2023), we index locations by their competition index $C$ rather than by their appeal $c$.[57] In doing so, we momentarily consider the inverse function $c(C)$. The profit function (3) takes the form:

$$\log\Pi(z,C) = \log M(C) + \log\psi\left(\frac{C}{z}\right) + \log\varphi\left(\psi\left(\frac{C}{z}\right)\right) - \log\frac{P(C)}{D(C)} + \log\left(1 - \frac{C}{z}\frac{1}{\psi\left(\frac{C}{z}\right)}\right), \quad \text{(B1)}$$

where substituted in the optimal relative prices and quantities from the profit maximization problem. Under this alternative formulation, firms sort based on the competition index $C$ rather than the appeal of a city $c$. To prove that there is stricly increasing assignment between $C$ and $z$, we

[57] See Bilal (2023), Appendix B.3.3.

use the methods of standard assignment problems (i.e. Galichon (2016)). In particular, note that because of the envelope theorem,

$$\frac{\partial \log \Pi(z,C)}{\partial C} = \frac{M'(C)}{M(C)} - \frac{P'(C)}{P(C)} + \frac{D'(C)}{D} - \frac{1}{z\psi\left(\frac{C}{z}\right) - C}.$$

Moreover, replacing the expression from the first-order condition (13) implies:

$$\frac{\partial \log \Pi(z,C)}{\partial C} = \frac{M'(C)}{M(C)} - \frac{P'(C)}{P(C)} + \frac{D'(C)}{D} - \frac{\sigma\left(\psi\left(\frac{C}{z}\right)\right) - 1}{C}.$$

Thus,

$$\frac{\partial^2 \log \Pi(z,C)}{\partial z \partial C} = \frac{1}{z^2}\sigma'\left(\psi\left(\frac{C}{z}\right)\right)\underbrace{\psi'\left(\frac{C}{z}\right)}_{>0} > 0 \iff \sigma'(\cdot) > 0.$$

We see that the profit function is log-sumpermodular if and only if the elasticity of demand is increasing the relative price. Therefore, under Marshall's second law, which holds in the Klenow and Willis (2016) specification, there is a strictly assignment function between $z$ and $C$, $z(C)$.

**Step 2: general equilibrium objects.** We now derive expressions for the general equilibrium objects under the PAM between $z$ and $C$ result. First, using the tools from Costinot and Vogel (2010), the definition of the competition price index (8) implies:

$$z'(C)\frac{M_e g(z(C))}{f_C(C)} = \frac{1}{\Upsilon\left(\varphi\left(\psi\left(\frac{C}{z(C)}\right)\right)\right)} \tag{B2}$$

where $f_C(C)$ is the equilibrium density of $C$. With this expression, the ratio $P(C)/D(C)$ in (A1) is given by:

$$\frac{P(C)}{D(C)} = \frac{\Upsilon\left(\varphi\left(\psi\left(\frac{C}{z(C)}\right)\right)\right)}{\psi\left(\frac{C}{z}\right)\varphi\left(\psi\left(\frac{C}{z}\right)\right)}$$

To ease notation, I define $\delta(C/z(C))$ as the inverse of the ratio above:

$$\delta\left(\frac{C}{z(C)}\right) \equiv \frac{D(C)}{P(C)} = \frac{\psi\left(\frac{C}{z}\right)\varphi\left(\psi\left(\frac{C}{z}\right)\right)}{\Upsilon\left(\varphi\left(\psi\left(\frac{C}{z(C)}\right)\right)\right)}$$

Then, ideal price index $P(C)$ is:

$$P(C) = D(C)\delta\left(\frac{C}{z(C)}\right),$$
$$= \nu\frac{W(C)^{\beta}R(C)^{(1-\beta)}}{C}\delta\left(\frac{C}{z(C)}\right) \tag{B3}$$

On the other hand, labor labor supply (11) implies that market size, $M(C)$, is expressed as:

$$M(C) = \frac{\eta\overline{L}}{\overline{U}}\frac{b(C)^{\theta}W(C)^{1+\theta}}{P(C)^{\eta\theta}R(C)^{\alpha\theta}} \tag{B4}$$

The housing land market clearing condition is:

$$R(C)^{\phi} = \frac{\alpha}{\eta}\frac{M(C)}{R(C)} + (1-\gamma)\frac{Q^T(C)}{R(C)} + (1-\beta)\frac{M(C)}{\mu\left(\frac{C}{z(C)}\right)R(C)}, \tag{B5}$$

where the terms on the right-hand side correspond to worker's, traded good producers, and local producers total housing consumption, respectively. Using the labor input demands, local labor market clearing implies:

$$L(C) = \beta\frac{M(C)}{W(C)\mu\left(\frac{C}{z(C)}\right)} + \gamma\frac{Q^T(C)}{W(C)}.$$

Using the definition of $M(C)$ and solving for the traded-good production gives:

$$Q^T(C) = \frac{M(C)}{\gamma}\left[\frac{1}{\eta} - \frac{\beta}{\mu\left(\frac{C}{z(C)}\right)}\right] \tag{B6}$$

Therefore, replacing the above expression for $Q^T(C)$ into (B5) gives:

$$R(C)^{\phi} = \frac{\alpha}{\eta}\frac{M(C)}{R(C)} + \frac{(1-\gamma)}{\gamma}\frac{M(C)}{R(C)}\left[\frac{1}{\eta} - \frac{\beta}{\mu\left(\frac{C}{z(C)}\right)}\right] + \frac{1-\beta}{\mu\left(\frac{C}{z(C)}\right)}\frac{M(C)}{R(C)}$$

Solving for $R(C)$ gives the equilibrium land rents:

$$R(C) = M(C)^{\frac{1}{1+\phi}}\chi\left(\frac{C}{z(C)}\right)^{\frac{1}{1+\phi}}, \tag{B7}$$

with

$$\chi\left(\frac{C}{z(C)}\right) \equiv \left[\frac{\alpha}{\eta} + \frac{(1-\gamma)}{\eta\gamma} + \frac{(1-\beta)\gamma - \beta(1-\gamma)}{\gamma\mu\left(\frac{C}{z(C)}\right)}\right]$$

The zero profit condition of the traded good producers imply that equilibrium wages are given by:

$$W(C) = \left(\frac{a(C)}{R(C)^{(1-\gamma)}\varrho}\right)^{\frac{1}{\gamma}}. \tag{B8}$$

Substituting (B7), (B3), and (B8) into the market size expression (B4) and solving for $M(C)$ gives:

$$M(C) = \left[\frac{\eta\overline{L}}{\overline{U}\nu^{\eta\theta}} \frac{C}{\delta\left(\frac{C}{z(C)}\right)\chi\left(\frac{C}{z(C)}\right)^{\xi-1}} a(C)^{\frac{1+\theta(1-\eta\beta)}{\gamma}} b(C)^{\theta}\right]^{\frac{1}{\xi}}, \tag{B9}$$

where $\xi \equiv \frac{\gamma(1+\phi+\theta(\alpha+\eta(1-\beta)))+(1-\gamma)(1+\theta(1-\eta\beta))}{\gamma(1+\phi)}$.[58]

**Step 3: single index property.** The assignment function $z(C)$ and the competition index $C$ are jointly determined by a coupled ODE system:

$$\begin{aligned} z'(C)\frac{M_e g(z(C))}{f_C(C)} &= \frac{1}{\Upsilon\left(\varphi\left(\psi\left(\frac{C}{z(C)}\right)\right)\right)}, \\ \sigma\left(\psi\left(\frac{C}{z(C)}\right)\right) - 1 &= \mathcal{E}_\delta(C) + \mathcal{E}_M(C), \end{aligned} \tag{B10}$$

where the first equation comes from (B2) (definition of competition price index) and the second from the first-order condition of the firm's location problem (B1). The first equation only depends on demand parameters, densities, $z(C)$ and $C$. On the other hand, to inspect the second equation, we look at (B2) and (B9). The the term on the left-hand side and the first term on the right-hand side depends on primitives of the model, $z(C)$ and $C$. On the contrary, the last term on the right-hand side, depends on primitives of the model, $z(C)$, $C$ and the combined object, $a(C)^{\frac{1+\theta(1-\eta\beta)}{\gamma}} b(C)^{\theta}$. Therefore, the equilibrium objects $z(C)$ and $C$ depend only on the characteristics of a location through a combined index with specific weights on productivity and amenities. This result implies that, local good producers will make their sorting decisions based on a unidimensional index $c(a,b)$ rather than considering the two dimensions of location heterogeneity separately:

[58] Note that $\xi - 1 = \frac{\gamma\theta(\alpha+\eta(1-\beta))+(1-\gamma)(1+\theta(1-\eta\beta))}{\gamma(1+\phi)} > 0$.

$$c = c(a,b) \equiv a^{\frac{1+\theta(1-\eta\beta)}{\gamma}} b^{\theta}$$

**Step 4: sorting in general equilibrium.** In the last step we characterize under which conditions more appealing locations have higher competition: i.e. the function $c(C)$ is increasing in general equilibrium. We can use (B9) to write the first-order condition (B10) as:

$$\begin{aligned}\sigma\left(\psi\left(\frac{C}{z(C)}\right)\right) - 1 &= \mathcal{E}_{\delta}(C) + \frac{1}{\xi}\left[1 + \mathcal{E}_{c}(C) - \mathcal{E}_{\delta}(C) - (\xi-1)\mathcal{E}_{\chi}(C)\right], \\ &= \frac{\xi-1}{\xi}\left[\mathcal{E}_{\delta}(C) - \mathcal{E}_{\chi}(C)\right] + \frac{1}{\xi}\left[1 + \mathcal{E}_{c}(C)\right]\end{aligned}$$

Under the Klenow and Willis (2016) specification, when $\varepsilon = 0$, $\delta(\cdot)$ and $\chi(\cdot)$ become constants and hence the above equation collapses to:

$$\overline{\sigma} - 1 = \frac{1}{\xi}\left[1 + \mathcal{E}_{c}(C)\right],$$

which implies that $\mathcal{E}_{c}(C) > 0$ if $\xi\left(\overline{\sigma} - 1\right) > 1$. Therefore, when $\xi\left(\overline{\sigma} - 1\right) > 1$, there exists a region of the space parameter where $\varepsilon$ is small and PAM is obtained in general equilibrium: both $z(C)$ and $c(C)$ are strictly increasing, and therefore more productive firms locate in more appealing cities.

## B.2 Proposition 2

Using the elasticity notation, (28) implies

$$\mathcal{E}_{\mathcal{M}}(c) = \mathcal{E}_{\mu}\left(\frac{\mathbb{C}(c)}{z(c)}\right)\left[\mathcal{E}_{C}(c) - \mathcal{E}_{z}(c)\right]$$

Recall that $\mu(\cdot)$ is a decreasing function, and hence, $\mathcal{E}_{\mu}\left(\frac{\mathbb{C}(c)}{z(c)}\right) < 0$. Therefore, we get that

$$\mathcal{E}_{\mathcal{M}}(c) < (>)\, 0 \iff \mathcal{E}_{C}(c) > (<)\, \mathcal{E}_{z}(c)$$

Equation (B13) shows that:

$$\begin{aligned}&\mathcal{E}_{C}(c)\left[\alpha\left(\sigma\left(\frac{\mathbb{C}(c)}{z(c)}\right) - 1\right) + (\alpha+\eta)\Theta\left(\frac{\mathbb{C}(c)}{z(c)}\right) + \frac{\xi-1}{\xi}\Lambda\left(\frac{\mathbb{C}(c)}{z(c)}\right) - \eta\theta\right] \\ &\quad = \xi + \left[(\alpha+\eta)\Theta\left(\frac{\mathbb{C}(c)}{z(c)}\right) + \frac{\xi-1}{\xi}\Lambda\left(\frac{\mathbb{C}(c)}{z(c)}\right)\right]\mathcal{E}_{z}(c),\end{aligned}$$

Then, solving for $\mathcal{E}_C(c)$ in the above equation yields that:

$$\mathcal{E}_z(c) > (<)\, \mathcal{E}_C(c) > \iff \mathcal{E}_z(c) > (<) \frac{\xi}{\alpha\left(\sigma\left(\frac{\mathbb{C}(c)}{z(c)}\right) - 1\right) - \eta\theta}$$

Furthermore, equation (B11) implies the following expression for the productivity elasticity with respect city's appeal:

$$\mathcal{E}_z(c) = \frac{f(c)c}{M_e g(z(c))z(c)} \frac{1}{\Upsilon\left(\varphi\left(\psi\left(\frac{\mathbb{C}(c)}{z(c)}\right)\right)\right)}$$

Hence,

$$\begin{aligned}
\mathcal{E}_z(c) > (<)\, \mathcal{E}_C(c) > \iff & \frac{f(c)c}{M_e g(z(c))z(c)} \frac{1}{\Upsilon\left(\varphi\left(\psi\left(\frac{\mathbb{C}(c)}{z(c)}\right)\right)\right)} > (<) \frac{\xi}{\xi\left(\sigma\left(\frac{\mathbb{C}(c)}{z(c)}\right) - 1\right) - \eta\theta}, \\
\iff & \frac{1}{g(z(c))z(c)} > (<) \frac{\xi\Upsilon\left(\varphi\left(\psi\left(\frac{\mathbb{C}(c)}{z(c)}\right)\right)\right)}{\xi\left(\sigma\left(\frac{\mathbb{C}(c)}{z(c)}\right) - 1\right) - \eta\theta} \frac{M_e}{f(c)c}
\end{aligned}$$

Moreover, under Assumption 2, the density-weighted productivity $g(z)z$ has the following closed form

$$g(z)z = \delta \frac{\left(\frac{z_L}{z}\right)^{\delta}}{1 - \left(\frac{z_L}{z_H}\right)^{\delta}}$$

With these expressions, we start by characterizing the conditions for the first inequality. Under the conditions of Proposition 1, we have that $\xi\left(\sigma\left(\frac{\mathbb{C}(c)}{z(c)}\right) - 1\right) - \eta\theta > 1 - \eta\theta$. Also, the parametric specification for $\Upsilon(\cdot)$ implies that $\Upsilon\left(\varphi\left(\psi\left(\frac{\mathbb{C}(c)}{z(c)}\right)\right)\right) < \overline{\Upsilon}$, where $\overline{\Upsilon}$ is a constant that depends on $\varepsilon$ and $\overline{\sigma}$. Recall that both traded good productivities and local amenities are defined over a bounded space. Therefore, there exists $\underline{c}$ such that $c > \underline{c}$ for all $c$. Finally, let $\underline{f}$ be the lower bound of the city density, which is exogenous. Thus, we obtain that $\mathcal{E}_z(c) > \mathcal{E}_C(c)$ if

$$\delta < \underbrace{\frac{(1 - \eta\theta)\, \underline{f}\underline{c}}{\xi\overline{\Upsilon}}}_{\equiv \overline{\delta}} \frac{1}{M_e}$$

With a sufficiently low entry cost, the term $1/M_e$ is large. Therefore, we conclude that if $\delta < \overline{\delta}$ and the entry cost is not too large, $\mathcal{E}_z(c) > \mathcal{E}_C(c)$ and hence city aggregate markup is increasing

in city appeal $c$.

## B.3 Proposition 3

The proof of Proposition 3 is structured in three steps. First, derive the system of ODEs that determine the equilibrium for the local good sector. Second. show existence of a solution to these systems conditional on general equilibrium aggregates, $s$ and $\overline{U}$. Third, show existence and uniqueness of general equilibrium objects.

**Step 1: ODE system for local goods producers.** Impose Assumption 1, and the assumptions of Proposition 1. Then, PAM between firm and location productivity is obtained. The definition of the competition price index (8) implies:

$$z'(c) = \frac{M_e f(c)}{g(z(c))} \frac{1}{\Upsilon\left(\varphi\left(\psi\left(\frac{\mathbb{C}(c)}{z(c)}\right)\right)\right)} \tag{B11}$$

The FOC of the location problem (21) is:

$$\mathcal{E}_{\mathbb{C}}(c) = \left[\mu\left(\psi\left(\frac{\mathbb{C}(c)}{z(c)}\right)\right) - 1\right][\mathcal{E}_{\delta}(c) + \mathcal{E}_{M}(c)] \tag{B12}$$

Equation (A1) implies:

$$\mathcal{E}_{\delta}(c) = \Theta\left(\frac{\mathbb{C}(c)}{z(c)}\right)\mathcal{E}_{z}(c) - \Theta\left(\frac{\mathbb{C}(c)}{z(c)}\right)\mathcal{E}_{\mathbb{C}}(c),$$

with $\Theta\left(\frac{\mathbb{C}(c)}{z(c)}\right) = \rho\left(\frac{\mathbb{C}(c)}{z(c)}\right)\left[\sigma\left(\psi\left(\frac{\mathbb{C}(c)}{z(c)}\right)\right) - 1\right]\left[\mu\left(\frac{\mathbb{C}(c)}{z(c)}\right)/\delta\left(\frac{\mathbb{C}(c)}{z(c)}\right) - 1\right]$. On the other hand, (B9) gives:

$$\mathcal{E}_{M}(c) = \frac{\eta\theta}{\xi}\mathcal{E}_{\mathbb{C}}(c) + \frac{\eta\theta}{\xi}\mathcal{E}_{\delta}(c) - \frac{\xi-1}{\xi}\Lambda\left(\frac{\mathbb{C}(c)}{z(c)}\right)[\mathcal{E}_{\mathbb{C}}(c) - \mathcal{E}_{z}(c)] + 1,$$

$$\mathcal{E}_{M}(c) = \frac{\eta\theta}{\xi}\mathcal{E}_{\mathbb{C}}(c) + \frac{\eta\theta}{\xi}\mathcal{E}_{\delta}(c) - \frac{\xi-1}{\xi}\mathcal{E}_{\chi}(c) + 1,$$

The definition of $\chi(\cdot)$ implies:

$$\Lambda\left(\frac{\mathbb{C}(c)}{z(c)}\right) \equiv \frac{\left(1 - \rho\left(\frac{\mathbb{C}(c)}{z(c)}\right)\right)}{\left(\alpha + \frac{1-\gamma}{\gamma}\right)\mu\left(\frac{\mathbb{C}(c)}{z(c)}\right) + \eta\left((1-\beta) - \frac{\beta(1-\gamma)}{\gamma}\right)}$$

Moreover, the definition of the competition index further implies that $\mathcal{E}_{\mathbb{D}}(c) = \beta\mathcal{E}_{W}(c) + (1-\beta) - \mathcal{E}_{R}(c) - \mathcal{E}_{\mathbb{C}}(c)$. Finally, the elasticity of wages $\mathcal{E}_{W}(c)$ is given by (22). Combining these conditions

into the location FOC, gives:

$$\frac{C'(c)c}{\mathbb{C}(c)}\left[\alpha\left(\xi\left(\frac{\mathbb{C}(c)}{z(c)}\right)-1\right)+(\alpha+\eta)\Theta\left(\frac{\mathbb{C}(c)}{z(c)}\right)+\frac{\xi-1}{\xi}\Lambda\left(\frac{\mathbb{C}(c)}{z(c)}\right)-\eta\theta\right]$$
$$=\xi+\left[(\alpha+\eta)\Theta\left(\frac{\mathbb{C}(c)}{z(c)}\right)+\frac{\xi-1}{\xi}\Lambda\left(\frac{\mathbb{C}(c)}{z(c)}\right)\right]\frac{z'(c)a}{z(c)}, \tag{B13}$$

where we explicitly wrote the expression for the elasticities $\mathcal{E}_z(c)$ and $\mathcal{E}_{\mathbb{C}}(c)$. Equations (B11) - (B13) define a coupled system of ODE's, with two boundary conditions $z(\overline{c})=\overline{z}$ and $z(\underline{c})=\underline{c}$. The first boundary condition states that the most productive firms go to the most appealing cities locations, while the second condition implies that least productive firms locate in tge least appealing cities. The solution to this ODE system is the assignemnt function $z(c)$, the competition index function $\mathbb{C}(c)$, and the market size function $M(c)$ that determine the equilibrium in the local sector, given $M_e$ and $\overline{U}$.

**Step 2: Existence of a solution to the ODE system given $M_e$ and $\overline{U}$.** Inspection of the the system (B11) - (B13) indicate that the systems satisfies standard regularity conditions for a unique solution given the general equilibrium objects $M_e$ and $\overline{U}$. In particular, the system is Lipschitz continuous. In particular, there exists $\underline{C}(M_e,\overline{U})$ such that $z(\underline{C})=\underline{z}$.[59]

**Step 3: Existence of $M_e$ and $\overline{U}$ and uniqueness** On the one hand, aggregate labor market clearing condition uniquely pins down $\overline{U}$:

$$\int_c L(c)f(c)dc=\overline{L} \qquad \longrightarrow \qquad \overline{U}=\left[\int_{\underline{c}}^{\overline{c}}\left(\frac{b(c)W(c)}{\mathbb{P}(c)^{\eta}R(c)^{\alpha}}\right)f(c)c\right]^{\frac{1}{\theta}}$$

On the other hand, $M_e$ is pin down by the traded-good market clearing condition (31):

$$c_eM_e=\int_c\left[Q^T(c)-L(c)Q(c)-\frac{\phi}{1+\phi}R(c)^{1+\phi}\right]f(c)dc$$

Note that te LHS of the above expression is an increasing function of $M_e$. Now suppose that the supports of $F_c$ and $G_z$ are small enough. This assumption makes posible using a first-order approximation of the the equilibrium expressions for $Q^T(c)$, $L(c)Q(c)$ and $R(c)$ in (B6), (6), and (B5). These approximations imply that the RHS a decreasing function of $M_e$. Therefore, there exists a unique $M_e$ such that the traded good market clearing condition is satisfied.

# C Theoretical Extensions

This section considers theoretical extensions to the baseline model.

[59] See Appendix B.5 in Bilal (2023) for a detailed discussion of the regularity conditions that guarantee a solution to these type of systems.

## C.1 General Kimball Aggregators

In this section, I extend the model predictions from the Klenow and Willis (2016) specification to general parametrization of the Kimball aggregator.

## C.2 Model with H.S.A. Preferences

In this section, I extend the theoretical results of Section 2.

# D Efficiency

## D.1 Social Planner's Problem

An utilitarian planner aims to maximize the sum of the utility levels of all the individuals in the economy. For the consumption side, the planner chooses traded good, $Q(c)$, local good, $Y(c)$, and housing consumption, $H(c)$, for every location. For the production side, the planner chooses the number of workers in the traded and local good sectors, $L^T(c)$ and $L^{NT}(c)$, which determine total population, $L(c) = L(c)^T + L^{NT}(c)$. Moreover, she also chooses materials, energy, and capital total usage in both sectors, $M^T(c)$, $E^T(c)$, $K^T(c)$, $M^{NT}(c)$, $E^{NT}(c)$, and $K^{NT}(c)$. For the location of local goods producers, I anticipate that the planner chooses PAM. Hence, she chooses the matching function $z(c)$, the slope of this function, and the mass of entrants $M_e$. To simplify the derivations, I follow an alternative formulation in which instead of choosing directly the slope of the assignment function, the planner chooses $\zeta(c)$, where $\zeta(c) \equiv \frac{z'(c)g(z(c))}{f(c)}$.[60] Finally, the planner also chooses the exit cutoff $z^*$ such that $z(\overline{a}) = \overline{z}$ and $z(\underline{a}) = z^*$. The planner therefore maximizes the objective function:

$$\Omega = \int_{\underline{a}}^{\overline{a}} \left(L^T(c) + L^{NT}(c)\right) \left(\frac{Y(c)}{\eta}\right)^{\eta} \left(\frac{H(c)}{\alpha}\right)^{\alpha} \left(\frac{Q(c)}{1-\eta-\alpha}\right)^{1-\eta-\alpha} f(c)da,$$

subject to the constraints:

[60] Note that with $z(c)$ and $\zeta(c)$, one can recover the true slope of the assignment function $z'(c) = \zeta(c)f(c)/g(z(c))$.

$$\int_{\underline{a}}^{\overline{a}} \left(L^T(c) + L^{NT}(c)\right) f(c)da = \overline{L},$$

$$\int_{\underline{a}}^{\overline{a}} a(L^T(c))^{\gamma}(M^T(c))^{\gamma_m}(E^T(c))^{\gamma_e}(K^T(c))^{\gamma_k} f(c)da = c_e M_e + \int_{\underline{a}}^{\overline{a}} \left\{Q(c)\left(L^T(c) + L^{NT}(c)\right)\right\} f(c)da$$

$$+ \int_{\underline{a}}^{\overline{a}} \left\{\Omega^e\left(E^T(c) + E^{NT}(c)\right) + M^T(c) + M^{NT}(c) + \Omega^k\left(K^T(c) + K^{NT}(c)\right)\right\} f(c)da$$

$$H(c)\left(L^T(c) + L^{NT}(c)\right) = 1 \; \forall \, a$$

$$\Upsilon\left(\frac{z(c)(L^{NT}(c))^{\beta}(M^{NT}(c))^{\beta_m}(E^{NT}(c))^{\beta_e}(K^{NT}(c))^{\beta_k}}{M_e\zeta(c)\left(L^T(c) + L^{NT}(c)\right)Y(c)}\right) M_e\zeta(c) = 1 \; \forall \, i \in a, \; \forall \, a$$

$$\int_{\underline{a}}^{a} \zeta(x)f(x)dx = G(z(c)) \; \forall \, a$$

$$\left(\frac{Y(c)}{\eta}\right)^{\eta}\left(\frac{H(c)}{\alpha}\right)^{\alpha}\left(\frac{Q(c)}{1-\eta-\alpha}\right)^{1-\eta-\alpha} = \overline{U} \; \forall \, i \in a, \; \forall \, a$$

The first constraint corresponds to the aggregate labor market clearing. The second constraint is the aggregate resource constraint. The third constraint states that the land markets clear in every location. The fourth constraint corresponds to the local resource constraint, coming from the local goods Kimball preferences. The fifth constraint is an adding up condition coming from the definition of $\zeta(c)$, and the last constraint corresponds to the free mobility condition. The planner's Lagrangian is given by:

$$
\begin{aligned}
\mathcal{L}^P = & \int_{\underline{a}}^{\overline{a}} \left(L^T(c) + L^{NT}(c)\right) \left(\frac{Y(c)}{\eta}\right)^{\eta} \left(\frac{H(c)}{\alpha}\right)^{\alpha} \left(\frac{Q(c)}{1-\eta-\alpha}\right)^{1-\eta-\alpha} f(c)da, \\
& + \varkappa_1 \left[\overline{L} - \int_{\underline{a}}^{\overline{a}} \left(L^T(c) + L^{NT}(c)\right) f(c)da\right] \\
& + \varkappa_2 \left[\int_{\underline{a}}^{\overline{a}} a(L^T(c))^{\gamma}(M^T(c))^{\gamma_m}(E^T(c))^{\gamma_e}(K^T(c))^{\gamma_k} f(c)da - c_e M_e - f M_e(1-G(z^*))\right. \\
& \quad \left. - \int_{\underline{a}}^{\overline{a}} \left\{Q(c)\left(L^T(c) + L^{NT}(c)\right) + M^T(c) + M^{NT}(c) + \Omega^e\left(E^T(c) + E^{NT}(c)\right) + \Omega^k\left(K^T(c) + K^{NT}(c)\right)\right\} f(c)d\right. \\
& + \int_{\underline{a}}^{\overline{a}} \varsigma_1(c)\left[1 - H(c)\left(L^T(c) + L^{NT}(c)\right)\right] f(c)da \\
& + \int_{\underline{a}}^{\overline{a}} \varsigma_2(c)\left[\Upsilon\left(\frac{z(c)(L^{NT}(c))^{\beta}(M^{NT}(c))^{\beta_m}(E^{NT}(c))^{\beta_e}(K^{NT}(c))^{\beta_k}}{M_e\zeta(c)\left(L^T(c) + L^{NT}(c)\right)Y(c)}\right) M_e\zeta(c) - 1\right] f(c)da \\
& - \int_{\underline{a}}^{\overline{a}} \varsigma_3(c)G(z(c))f(c)da + \int_{\underline{a}}^{\overline{a}} (\overline{\varsigma}_3 - \vartheta(c))\zeta(c)f(c)da, \\
& + \int_{\underline{a}}^{\overline{a}} \varsigma_4(c)\left[\overline{U} - \left(\frac{Y(c)}{\eta}\right)^{\eta}\left(\frac{H(c)}{\alpha}\right)^{\alpha}\left(\frac{Q(c)}{1-\eta-\alpha}\right)^{1-\eta-\alpha}\right] f(c)da
\end{aligned}
$$

where $\varkappa_1$, $\varkappa_2$, $\varsigma_1(c)$, $\varsigma_2(c)$, $\varsigma_3(c)$, and $\varsigma_4(c)$ are lagrange multipliers.[61] I integrated by parts the constraint on $\zeta(c)$ with multiplier $\varsigma_3(c)$, and defined $\overline{\varsigma}_3 \equiv \int_{\underline{a}}^{\overline{a}} \varsigma_3(c)f(c)da$ and $\vartheta(c) \equiv \int_{\underline{a}}^{a} \varsigma_3(x)f(x)dx$.

**Consumption and housing.** The first-order conditions with respect to $Y(c)$, $H(c)$, and $Q(c)$ are respectively:

$$
\left[1 - \frac{\varsigma_4(c)}{L(c)}\right] \eta \frac{U(c)}{Y(c)} = \varsigma_2(c) \frac{\Upsilon'(q(c))q(c)}{Y(c)} \zeta(c), \tag{D1}
$$

$$
\left[1 - \frac{\varsigma_4(c)}{L(c)}\right] \alpha \frac{U(c)}{H(c)} = \varsigma_1(c), \tag{D2}
$$

$$
\left[1 - \frac{\varsigma_4(c)}{L(c)}\right] (1-\eta-\alpha) \frac{U(c)}{Q(c)} = \varkappa_2, \tag{D3}
$$

where $U(c) \equiv \left(\frac{Y(c)}{\eta}\right)^{\eta}\left(\frac{H(c)}{\alpha}\right)^{\alpha}\left(\frac{Q(c)}{1-\eta-\alpha}\right)^{1-\eta-\alpha}$, and $q(c) \equiv \frac{z(c)(L^{NT}(c))^{\beta}(M^{NT}(c))^{\beta_m}(E^{NT}(c))^{\beta_e}(K^{NT}(c))^{\beta_k}}{\zeta(c)L(c)Y(c)}$, with $L(c) \equiv L^T(c) + L^{NT}(c)$. Normalizing $\varkappa_2$ to be one, these first-order conditions imply that:

[61] I can shut down the free mobility constraint by setting $\varsigma_4(c) = 0$ for all $a$.

$$\begin{aligned}\mathbb{P}(c)Y(c) &= \eta\overline{U}\mathbb{P}(c)^{\eta}\varsigma_1(c)^{\alpha},\\ \varsigma_1(c)H(c) &= \alpha\overline{U}\mathbb{P}(c)^{\eta}\varsigma_1(c)^{\alpha},\\ Q(c) &= (1-\eta-\alpha)\overline{U}\mathbb{P}(c)^{\eta}\varsigma_1(c)^{\alpha},\end{aligned}$$

with:

$$\begin{aligned}\mathbb{P}(c) &\equiv \frac{\mathbb{D}(c)}{\delta(q(c))},\\ \mathbb{D}(c) &\equiv \varsigma_2(c)Y(c)L(c)\end{aligned}$$

Moreover, the ratio $\varsigma_4(c)/L(c)$ is given by:

$$\frac{\varsigma_4(c)}{L(c)} = 1 - \mathbb{P}(c)^{\eta}\varsigma_1(c)^{\alpha}$$

**Labor.** Define the planner's shadow wage $W^*(c)$ as:

$$W^*(c) = \varkappa_1 - \overline{U}\frac{\varsigma_4(c)}{L(c)} \tag{D4}$$

Then, the first-order conditions with respect to $L^T(c)$, and $L^{NT}(c)$ are respectively:

$$\gamma\frac{Q^T(c)}{L^T(c)} = W^*(c), \tag{D5}$$

$$\beta\frac{\mathbb{P}(c)Y(c)L(c)}{L^{NT}(c)} = W^*(c),, \tag{D6}$$

where $Q^T(c) \equiv a(L^T(c))^{\gamma}(M^T(c))^{\gamma_m}(E^T(c))^{\gamma_e}(K^T(c))^{\gamma_k}$, and where we used (D1) - (D3) for the definition of the planner's shadow wage, $W^*(c)$.

**Materials, energy and capital.** The first-order conditions with respect $M^T(c)$, $M^{NT}(c)$, $E^T(c)$, $E^{NT}(c)$, $K^T(c)$, and $L^{NT}(c)$ are respectively:

$$\gamma_m \frac{Q^T(c)}{M^T(c)} = 1,$$
$$\beta_m \frac{\mathbb{P}(c)Y(c)L(c)}{M^{NT}(c)} = 1,$$
$$\gamma_e \frac{Q^T(c)}{E^T(c)} = \Omega^e,$$
$$\beta_e \frac{\mathbb{P}(c)Y(c)L(c)}{E^{NT}(c)} = 1\Omega^e,$$
$$\gamma_K \frac{Q^T(c)}{K^T(c)} = \Omega^k,$$
$$\beta_k \frac{\mathbb{P}(c)Y(c)L(c)}{K^{NT}(c)} = 1\Omega^k$$

Combining (D5) with the rest of the traded good inputs equilibrium conditions gives the planner's counterpart of (22):

$$W^*(c) = \left(\frac{a}{\varrho}\right)^{\frac{1}{\gamma}}$$

Similarly, combining (D4) with the rest of the non-traded good inputs equilibrium conditions gives the optimal condition for $q^*(c)$:

$$\Upsilon'(q^*(c)) = \frac{C^*(c)}{z(c)}, \tag{D7}$$

where $C^*(c)$ is the planner's competition index given by:

$$C^*(c) \equiv \nu \frac{(W^*(c))^\beta}{D^*(c)}.$$

**Allocation of non-traded varieties producers.** The first-order conditions with respect $z(c)$ and $\zeta(c)$ are respectively:

$$\varsigma_2(c) \frac{\Upsilon'(q(c))q(c)\zeta(c)}{z(c)} = \varsigma_3(c)g(z(c)),$$
$$\varsigma_2(c)\left[\Upsilon(q(c)) - \Upsilon'(q(c))q(c)\right] = \vartheta(c) - \overline{\varsigma}_3$$

Define $J(c) \equiv \vartheta(c) - \overline{\varsigma}_3$. Therefore, we have that $J'(c) = \varsigma_3(c)f(c)$ and we can re-write the first-order condition with respect to $z(c)$ as:

$$\varsigma_2(c)\frac{\Upsilon'(q(c))q(c)\zeta(c)}{z(c)} = \frac{J'(c)}{J(c)}(\vartheta(c) - \overline{\varsigma}_3)\frac{g(z(c))}{f(c)}$$

We can combine the above equation with the first-order condition with respect to $\zeta(c)$ to get:

$$\mathcal{E}_J(c) = \frac{\mathcal{E}_z(c)}{\delta(q(c)) - 1}, \tag{D8}$$

where we used the definition of $\zeta(c)$ and $\delta(q(c))$. Furthermore, re-write the first-order condition with respect to $\zeta(c)$ as:

$$J(c) = D^*(c)L(c)Y(c)\Upsilon(q(c))\frac{\delta(q(c)) - 1}{\delta(q(c))}$$

Then,

$$\mathcal{E}_J(c) = \mathcal{E}_{D^*}(c) + \mathcal{E}_L(c) + \mathcal{E}_Y(c) + \mathcal{E}_q(c)\left[\frac{1}{\delta(q(c))} + \frac{\mathcal{E}_\delta(q(c))}{\delta(q(c)) - 1}\right]$$

The elasticities $\mathcal{E}_\delta(q(c))$ and $\mathcal{E}_q(c)$ are given by:

$$\mathcal{E}_\delta(q(c)) = \frac{1}{\delta(q(c))} + \frac{1}{\sigma(q(c))} - 1, \tag{D9}$$

$$\mathcal{E}_{q^*}(c) = \sigma(q(c))\left(\mathcal{E}_z(c) - \mathcal{E}_{C^*}(c)\right) \tag{D10}$$

where we used the definition of $\delta(q(c))$ for the first expression and (D7) for the second. Define $\Theta^*(c)$ as:

$$\Theta^*(c) \equiv (\sigma(q(c)) - 1)\left(\frac{\mu(q(c))}{\delta(q(c))} - 1\right).$$

With this notation:

$$\begin{aligned}
\mathcal{E}_{\delta^*}(c) &= \mathcal{E}_\delta(q(c))\mathcal{E}_{q^*}(c), \\
&= \mathcal{E}_\delta(q(c))\sigma(q(c))\left(\mathcal{E}_z(c) - \mathcal{E}_{C^*}(c)\right), \\
&= (\sigma(q(c)) - 1)\left(\frac{\mu(q(c))}{\delta(q(c))} - 1\right)\left(\mathcal{E}_z(c) - \mathcal{E}_{C^*}(c)\right), \\
&= \Theta^*(c)\left(\mathcal{E}_z(c) - \mathcal{E}_{C^*}(c)\right),
\end{aligned}$$

Combining these expressions gives:

$$\mathcal{E}_J(c) = \mathcal{E}_{D^*}(c) + \mathcal{E}_L(c) + \mathcal{E}_Y(c) + \frac{\mathcal{E}_z(c)}{\delta(q(c)) - 1} - \frac{\mathcal{E}_{C^*}(c)}{\delta(q(c)) - 1} \tag{D11}$$

Equating (D8) and (D11) implies:

$$\mathcal{E}_{C^*}(c) = (\delta(q(c)) - 1)\left[\mathcal{E}_{\delta^*}(c) + \mathcal{E}_{M^*}(c)\right], \tag{D12}$$

where $M^*(c) \equiv P^*(c)Y(c)L(c)$ is the planner's market size. This expression closely resembles the one for the decentralized equilibrium (B12). To find an expression for the planner's market size $M^*(c)$, first note that land market clearing implies:

$$L(c) = \frac{\varsigma_1(c)}{\alpha \overline{U} \mathbb{P}(c)^\eta \varsigma_1(c)^\alpha} \tag{D13}$$

Replacing this expression into the optimal consumption decisions, we get that:

$$M^*(c) = \frac{\eta}{\alpha}\varsigma_1(c)$$

On the other hand, the definition of $W^*(c)$ implies that:

$$\varsigma_1(c) = \left[\frac{W^*(c) + \overline{U} - \varkappa_1}{\overline{U}\mathbb{P}(c)^\eta}\right]^{\frac{1}{\alpha}}$$

Denoting $\check{W}^*(c) \equiv W^*(c) + \overline{U} - \varkappa_1$, we obtain the following expression for the planner's market size:

$$\begin{aligned}\mathcal{E}_{M^*}(c) &= \frac{1}{\alpha}\left[\mathcal{E}_{\check{W}^*}(c) - \eta\mathcal{E}_{\mathbb{P}}(c)\right], \\ &= \frac{1}{\alpha}\left[\frac{W^*(c)}{\check{W}^*(c)}\frac{1}{\gamma} - \frac{\eta\beta}{\gamma} + \eta\mathcal{E}_{C^*}(c) + \eta\mathcal{E}_{\delta}(c)\right]\end{aligned}$$

Replacing the resulting expression into (D12) and solving for $\mathcal{E}_{C^*}(c)$ gives:

$$\mathcal{E}_{C^*}(c)\left(\alpha - \eta(\delta(q(c)) - 1)\right) = (\delta(q(c)) - 1)\left[\frac{1 - \eta\beta}{\gamma} + \Xi(q(c), \overline{U}, \varkappa_1)\right],$$

with

$$\Xi(q(c), \overline{U}, \varkappa_1) = (\alpha + \eta)\mathcal{E}_{\delta^*}(c) - \frac{1}{\gamma}\frac{\overline{U} - \varkappa_1}{W^*(c) + \overline{U} - \varkappa_1}$$

**Closing the social planner's problem.** Note that after solving the planner's ODE's we ge $C^*(c)$ and $z(c)$. This pins down $q^*(c)$ by (D7), $\mathbb{D}(c)$ and $\mathbb{P}(c)$.

Inputs first-order conditions imply that:

$$\mathbb{P}(c)Y(c)L(c) + Q^T(c) = W(c)L(c) + M(c) + \Omega^e E(c) + \Omega^e E(c)$$

Therefore, the aggregate resource constraint becomes:

$$\int_{\underline{a}}^{\overline{a}} \left[\mathbb{P}(c)Y(c)L(c) + Q(c)L(c)\right] f(c)da = \int_{\underline{a}}^{\overline{a}} W(c)L(c)f(c)da$$

This condition states that, in the aggregate, traded and non-traded goods total revenue is equal to the aggregate wage-bill. Combining (D13) with the expressions for $Y(c)$ and $Q(c)$ we get that $\mathbb{P}(c)Y(c)L(c) = (\eta/\alpha)\varsigma_1(c)$ and $Q(c)L(c) = ((1-\eta-\alpha)/\alpha)\varsigma_1(c)$. Therefore:

$$\int_{\underline{a}}^{\overline{a}} \frac{1-\alpha}{\alpha}\varsigma_1(c)f(c)da = \int_{\underline{a}}^{\overline{a}} W(c)L(c)f(c)da$$

Which using the equilibrium conditions can be written as:

$$(1-\alpha)\int_{\underline{a}}^{\overline{a}} \left[\frac{W^*(c) + \overline{U} - \varkappa_1}{\mathbb{P}(c)^\eta}\right]^{\frac{1}{\alpha}} f(c)da = \int_{\underline{a}}^{\overline{a}} W(c) \left[\frac{\left(W^*(c) + \overline{U} - \varkappa_1\right)^{1-\alpha}}{\mathbb{P}(c)^\eta}\right]^{\frac{1}{\alpha}} f(c)da \tag{D14}$$

Finally, use (D13) and the aggregate labor market clearing to write:

$$\int_{\underline{a}}^{\overline{a}} \left[\frac{\left(W^*(c) + \overline{U} - \varkappa_1\right)^{1-\alpha}}{\mathbb{P}(c)^\eta}\right]^{\frac{1}{\alpha}} f(c)da = \overline{L} \tag{D15}$$

Equations (D14) and (D15) pin down $\overline{U}$ and $\varkappa_1$.

**Decentralized ODE.** The FOC of the location problem (21) is:

$$\mathcal{E}_{\mathbb{C}}(c) = \left[\mu\left(q(c)\right) - 1\right]\left(\mathcal{E}_\delta(c) + \mathcal{E}_M(c)\right)$$

With $\mathcal{E}_\delta(c) = \mathcal{E}_\delta(q(c))\mathcal{E}_q(c)$, where

$$\mathcal{E}_\delta(q(c)) = \frac{1}{\delta(q(c))} + \frac{1}{\sigma(q(c))} - 1, \tag{D16}$$

$$\mathcal{E}_q(c) = \rho(q(c))\sigma(q(c))\left(\mathcal{E}_z(c) - \mathcal{E}_{\mathbb{C}}(c)\right) \tag{D17}$$

Importantly, $\mathcal{E}_\delta(q(c))$ comes from the definition of the object $\delta(\cdot)$ and $\mathcal{E}_q(c)$ comes from the first-order condition of the firm. Therefore, defining:

$$\Theta(c) \equiv \rho(q(c))(\sigma(q(c)) - 1)\left(\frac{\mu(q(c))}{\delta(q(c))} - 1\right),$$

we get that:

$$\begin{aligned}
\mathcal{E}_\delta(c) &= \mathcal{E}_\delta(q(c))\mathcal{E}_q(c), \\
&= \mathcal{E}_\delta(q(c))\rho(q(c))\sigma(q(c))\left(\mathcal{E}_z(c) - \mathcal{E}_{\mathbb{C}}(c)\right), \\
&= \rho(q(c))(\sigma(q(c)) - 1)\left(\frac{\mu(q(c))}{\delta(q(c))} - 1\right)\left(\mathcal{E}_z(c) - \mathcal{E}_{\mathbb{C}}(c)\right), \\
&= \Theta(c)\left(\mathcal{E}_z(c) - \mathcal{E}_{\mathbb{C}}(c)\right),
\end{aligned}$$

Market size is given by:

$$\begin{aligned}
\mathcal{E}_M(c) &= \frac{1}{\alpha}\mathcal{E}_W(c) - \frac{\eta}{\alpha}\mathcal{E}_P(c), \\
&= \frac{1}{\alpha}\mathcal{E}_W(c) - \frac{\eta}{\alpha}\left(\mathcal{E}_D(c) - \mathcal{E}_\delta(c)\right), \\
&= \frac{1}{\alpha}\mathcal{E}_W(c) - \frac{\eta}{\alpha}\left(\beta\mathcal{E}_W(c) - \mathcal{E}_{\mathbb{C}}(c) - \mathcal{E}_\delta(c)\right), \\
&= \frac{1-\eta\beta}{\alpha}\mathcal{E}_W(c) + \frac{\eta}{\alpha}\mathcal{E}_{\mathbb{C}}(c) + \frac{\eta}{\alpha}\mathcal{E}_\delta(c)
\end{aligned}$$

Plugging the expressions for $\mathcal{E}_\delta(c)$ and $\mathcal{E}_M(c)$ into the first-order condition gives:

$$\mathcal{E}_{\mathbb{C}}(c) = \frac{\mu\left(q(c)\right) - 1}{\alpha + (\mu(q(c)) - 1)((\alpha+\eta)\Theta(c) - \eta)}\left[\frac{1-\eta\beta}{\gamma} + (\alpha+\eta)\Theta(c)\mathcal{E}_z(c)\right] \tag{D18}$$

## D.2 Misallocation from Increasing Markups

This section shows how misallocation of firms across space is exacerbated by increasing markups on city size.

### D.3 First-best Implementation

# E Markup Estimation

This section further explores the framework for markup estimation of Section 4.4.2.

## E.1 Main Derivations for Estimation

In this section, I derive additional results for the baseline markup estimation. Furthermore, it shows how to derive (49) under the Klenow and Willis (2016) functional form.

## E.2 Multi-Sector Estimation and Labor Market Power

In this section, I show how to extend the baseline estimation framework when having multiple sectors. Moreover, it shows how can we control for potential labor market power.

The multi-sector markup estimation procedure considers a framework in which consumers have Cobb-Douglas preferences over different bundles of local varieties within a sector. Formally, the bundle of local varieties $Y(c)$ is a Cobb-Douglas aggregator of sector-specific bundles:

$$Y(c) = \prod_{n=1}^{N} Y_n(c),$$

where $n$ denotes the sector. Furthermore, each of the sector bundles $Y_n(c)$ is implicitly defined by a Kimball aggregator:

$$\int_z \Upsilon_n \left( \frac{y_n(z,c)}{Y_n(c)} \right) dG_{n,c}(z) = 1,$$

where $y_n(z,c)$ is the consumption in city $c$ of a variety produced by a firm with productivity $z$ in sector $n$, $G_{n,c}(z)$ is the local productivity distribution of sector $n$ in city $c$, and the Kimball aggregator $\Upsilon_n(\cdot)$ now is sector-specific. Under this alternative formulation, all derivations from Section 4 extend, with the caveat that the markup function (43) is sector specific

$$\mu_{jnc} = \mu_n \left( \zeta_n \left( s_{jnc} \frac{P_{nc}}{D_{nc}} \right) \right), \tag{E1}$$

with markups, sales shares and price indices being sector specific as well. Moreover, under the multi-sector formulation, we allow different sectors to have different production functions

$$y_n(z,c) = z l_n(z,c)^{\beta_n} s_n(z,c)^{1-\beta_n}. \tag{E2}$$

Equations (E1) and (E2) imply the multi-sector estimating equation (46).

## E.3 General Production Function

This section derives the markup estimating equation with a general production. Because data in sectors other than Manufacturing is limited, I derive the estimation equation using the data available for Manufacturing. Formally, consider a Hicks-neutral production function in labor, materials, energy and capital:

$$y(z,c) = z\mathcal{F}\left(l(z,c), m(z,c), e(z,c), k(z,c)\right), \tag{E3}$$

where $\mathcal{F}(\cdot)$ is a continuously differentiable function, $m(z,c)$ denotes materials, $e(z,c)$ denotes energy and $k(z,c)$ denotes capital. Under this specification, the elasticities of output with respect to labor, materials and energy are given by

$$\begin{aligned}
\frac{\partial \log y(z,c)}{\partial \log l(z,c)} &= \frac{\partial \mathcal{F}\left(l(z,c), m(z,c), e(z,c), k(z,c)\right)}{\partial l(z,c)} \frac{l(z,c)}{\mathcal{F}\left(l(z,c), m(z,c), e(z,c), k(z,c)\right)} \\
\frac{\partial \log y(z,c)}{\partial \log m(z,c)} &= \frac{\partial \mathcal{F}\left(l(z,c), m(z,c), e(z,c), k(z,c)\right)}{\partial m(z,c)} \frac{m(z,c)}{\mathcal{F}\left(l(z,c), m(z,c), e(z,c), k(z,c)\right)}, \\
\frac{\partial \log y(z,c)}{\partial \log e(z,c)} &= \frac{\partial \mathcal{F}\left(l(z,c), m(z,c), e(z,c), k(z,c)\right)}{\partial e(z,c)} \frac{e(z,c)}{\mathcal{F}\left(l(z,c), m(z,c), e(z,c), k(z,c)\right)}.
\end{aligned}$$

Note that in all cases, the Hicks-neutral assumption on the production function implies that the output elasticity with respect to any variable input is just a function of the inputs of production:

$$\begin{aligned}
\frac{\partial \log y(z,c)}{\partial \log l(z,c)} &\equiv \kappa_l\left(l(z,c), m(z,c), e(z,c), k(z,c)\right), \\
\frac{\partial \log y(z,c)}{\partial \log m(z,c)} &\equiv \kappa_m\left(l(z,c), m(z,c), e(z,c), k(z,c)\right) \\
\frac{\partial \log y(z,c)}{\partial \log e(z,c)} &\equiv \kappa_e\left(l(z,c), m(z,c), e(z,c), k(z,c)\right).
\end{aligned}$$

Therefore, under the general production function (E3), the markup estimating equation (45) takes the form of:

$$\log \alpha^l_{jc} = \kappa_x\left(l(z,c), m(z,c), e(z,c), k(z,c)\right) - \varsigma_{1,c} s_{jc} - \varsigma_{2,c} s^2_{jc} - \varsigma_{3,c} s^3_{jc} - \upsilon_{jc}, \tag{E4}$$

where $x$ denotes the input of production, $x \in \{l, m, e\}$. The function $\kappa_x\left(l(z,c), m(z,c), e(z,c), k(z,c)\right)$ can be semi-parametric approximated as the markup function. Formally, I approximate this function by a third-order polynomial in its arguments as in Gandhi, Navarro, and Rivers (2020).

# F Model Inversion

Within the loop of the SMM estimation, I perform the model inversion to obtain local productivities and amenities. Given parameters, I solve the decentralized equilibrium and use (22) to recover traded good productivity

$$a(c) = \varrho W(c)^{\gamma} R(c)^{1-\gamma},$$

where $W(c)$ is data on the county average wages and $R(c)$ is the model's implied housing rent. Similarly, I obtain local productivities from the labor supply condition (11). Replacing the equilibrium objects of the model in such expression gives

$$b(c) = \frac{L(c)^{\frac{\lambda_1}{\theta}}}{W(c)_2^{\lambda}} \nu^{\eta} \left(\frac{\overline{U}}{\overline{L}}\right)^{\frac{1}{\theta}} \frac{\chi \left(\frac{\mathbb{C}(c)}{z(c)}\right)^{\frac{\alpha+\eta\beta}{1+\phi}}}{\left(\mathbb{C}(c)\delta \left(\frac{\mathbb{C}(c)}{z(c)}\right)\right)^{\eta}},$$

where $L(c)$ and $W(c)$ are data on counties population and average wages, and $\lambda_1$ and $\lambda_2$ are constants given by

$$\lambda_1 = 1 + \frac{\theta(\alpha + \eta\beta)}{1+\phi}, \qquad \text{and} \qquad \lambda_2 = \frac{(1+\phi)(1-\eta\beta) - (\alpha+\eta\beta)}{1+\phi}.$$

# G Additional Empirical Results

## G.1 Markups Across Cities for additional Sectors

Table G1 displays the estimated mean elasticity of county aggregate markup and county size for different 2-digit NAICS sectors. The establishment-level markup is estimated using equation (46). The results show heterogeneity across sectors in the estimated elasticity of county-sector aggregate markup and county size. Manufacturing, Wholesale, Transportation, Information and Education Services display a positive elasticity, while the other sectors display a negative elasticity. Strikingly, the elasticity for all sectors is statistically significant than zero, suggesting that there is indeed large variation in the degree of local competition across counties in all sectors.

The results also shed light on the sectors that are driving the aggregate negative relationship of markup and city size displayed in Figure 2. The bottom row in each panel show the average employment share of each sector among the total employment of local industries across counties. The sectors with higher local employment share are Retail, Healthcare, and Accommodation and Food Services. The three sectors display a negative elasticity, suggesting that they are the main drivers behind the results in Figure 2

Table G1: County Aggregate markup and County Size Regressions by Sector

| | | | **Panel A** | | |
|---|---|---|---|---|---|
| | Construction | Manufacturing | Wholesale | Retail | Transportation |
| | (1) | (2) | (3) | (4) | (5) |
| Log total labor income | -0.0427*** | 0.1496*** | 0.0817*** | -0.0706*** | 0.0448*** |
| | (0.0077) | (0.0131) | (0.0114) | (0.0059) | (0.0089) |
| Observations | 3100 | 1900 | 2400 | 3100 | 2800 |
| R-squared | 0.011 | 0.045 | 0.014 | 0.064 | 0.006 |
| Avg. Local Emp. Share | 0.0796 | 0.013 | 0.0203 | 0.241 | 0.0262 |
| | | | **Panel B** | | |
| | Information | Finance | Real Estate | PST Services | AWR Services |
| | (1) | (2) | (3) | (4) | (5) |
| Log total labor income | 0.1218*** | -0.0428*** | -0.0151* | -0.0565*** | -0.0157* |
| | (0.0099) | (0.0083) | (0.0086) | (0.0081) | (0.0089) |
| Observations | 2800 | 3100 | 2800 | 3000 | 2900 |
| R-squared | 0.035 | 0.009 | 0.001 | 0.018 | 0.001 |
| Avg. Local Emp. Share | 0.0162 | 0.0464 | 0.0157 | 0.0274 | 0.0553 |
| | | | **Panel C** | | |
| | Education Services | Healthcare | Arts, Entertainment and Recreation | Accommodation and Food Services | Other Services |
| | (1) | (2) | (3) | (4) | (5) |
| Log total labor income | 0.1384*** | -0.0639*** | 0.1567*** | -0.0718*** | -0.1115*** |
| | (0.0162) | (0.0097) | (0.0123) | (0.0073) | (0.0076) |
| Observations | 1400 | 3100 | 2100 | 3100 | 3100 |
| R-squared | 0.041 | 0.016 | 0.054 | 0.044 | 0.076 |
| Avg. Local Emp. Share | 0.00379 | 0.273 | 0.0112 | 0.143 | 0.05 |

**Notes:** Table G1 displays the average elasticity of county aggregate markup and county size. County aggregate markup is defined as (47) and county size is defined as total labor income. Dependent variable in all columns and panels is log county aggregate markup. County aggregate markup is computed using establishments in local industries within the specific sector. Sectors are defined as 2-digits NAICS sectors. PST services: Professional, Scientific, and Technical Services. ARW Services: Administrative and Support and Waste Management and Remediation Services. Average local employment share is the average employment share of the sector across counties among the total employment of local industries. Robust standard errors in parenthesis. *10% level, **5% level, ***1% level.

## G.2 Robustness Exercises

This section presents the robustness exercises for the empirical analysis section. To perform these exercises, I use Manufacturing as it is the only sector with detailed data on different inputs of production other than labor.

I estimate markups for Manufacturing using the equation (E4) derived in Appendix E.3. In particular, I use materials and energy as flexible inputs. As highlighted by Yeh, Macaluso, and Hershbein (2022), when there is labor market power, the markdown firms charge in the labor market will appear in the first-order condition for labor. Arguably, the materials and energy markets are such that firms do not have any market power in those input markets. Moreover, the data in Manufacturing also allow me to relax the assumption of constant output elasticities. Then, I follow Foster, Grim, and Haltiwanger (2016) and construct measures of labor, materials, energy, and capital usage at the establishment level. Equipped with these measures, I estimate markups using (E4), where a third-order polynomial in labor, materials, energy, and capital approximates the elasticities for materials and energy.

Estimating markups for Manufacturing using (E4) then serves for two robustness checks: 1) using an input for which producers do not have input market power, and 2) considering a general production function for which output elasticities are not constant and does not necessarily exhibit constant returns to scale.

On the one hand, Table G2 displays the results of regression between the baseline manufacturing markups estimated using (45) and the estimates using (E4). The baseline markups are estimated pooling all establishments in local industries while the estimates using (E4) only include establishments in local manufacturing industries. Moreover, I consider two definitions of a city: a county and a Commuting Zone. Columns indicate the baseline markups, whereas rows indicate the alternative estimates.

The results show that baseline markups highly correlate with the alternative estimates. Strikingly, the regression coefficients are close to one, suggesting that the baseline and the alternative markups move almost one-for-one. Nonetheless, the constant terms are positive and statistically significant than zero in all columns, suggesting that the baseline markups exhibit slightly higher levels than the alternative estimates.

On the other hand, Table (G3) shows the elasticity of county aggregate markup to county size using the alternative estimates. This table replicates the results in Figure 3(b). The elasticity of aggregate markup to county size remains almost unchanged when considering the alternative markup estimates. Indeed, the elasticity in Figure 3(b) is 0.14 while the elasticities reported in Table (G3) are 0.141 for materials and 0.135 for energy. The results are reassuring in two ways. First, the flexible polynomial that controls for potential market power in the baseline estimation indeed corrects for any potential monopsony power in the labor market. Second, the Cobb-Douglas technology assumption in the baseline estimation seems not restrictive as considering a more flexible production function yields similar county markup and size elasticities.

Table G2: Regressions baseline markups and extensions for Manufacturing

| | Baseline County (1) | Baseline CZ (2) | Baseline County (3) | Baseline CZ (4) |
|---|---|---|---|---|
| Flex. PF, Energy (County) | 1.036*** | | | |
| | (0.004) | | | |
| Flex. PF, Energy (CZ) | | 1.022*** | | |
| | | (0.005) | | |
| Flex. PF, Materials (County) | | | 1.075*** | |
| | | | (0.005) | |
| Flex. PF, Materials (CZ) | | | | 1.061*** |
| | | | | (0.005) |
| Constant | 0.341*** | 0.281*** | 0.304*** | 0.243*** |
| | (0.009) | (0.007) | (0.009) | (0.007) |
| Observations | 27500 | 27500 | 27500 | 27500 |
| R-squared | 0.745 | 0.782 | 0.738 | 0.778 |

**Notes:** Table G2 displays coefficients of a regression between the baseline markups and the alternative markup estimates for Manufacturing. The baseline markups are the ones estimated by equation (45) and pool establishments in all local industries. Alternative markup estimates are estimated using (E4) and consider only establishments in local manufacturing industries. Columns indicate the baseline markups and rows indicate the alternative estimates. Different columns and rows consider two definitions of a city: county and Commuting Zone (CZ). Robust standard errors in parenthesis. *10% level, **5% level, ***1% level.

Table G3: County Aggregate Markup and County Size for Alternative Manufacturing Markups Estimates

| | Log agg. Markup Materials (1) | Log agg. Markup Energy (2) |
|---|---|---|
| Log labor income | 0.141*** | 0.135*** |
| | (0.006) | (0.006) |
| Observations | 1800 | 1800 |
| R-squared | 0.183 | 0.17 |

**Notes:** Table G3 displays the average elasticity of county aggregate markup and county size, using the alternative estimates for Manufacturing. Alternative markup estimates are estimated using (E4) and consider only establishments in local manufacturing industries. The dependent variable in Column (1) is the log county aggregate markup using materials as the flexible input in (E4). The dependent variable in Column (2) is the log county aggregate markup using energy as the flexible input in (E4). County size is defined as total labor income. Robust standard errors in parenthesis. *10% level, **5% level, ***1% level.